The Standard Model of particle physics and beyond.

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Outline.

1 Context.

2 Special relativity and gauge theories.
   - Action and symmetries.
   - Poincaré and Lorentz algebras and their representations.
   - Relativistic wave equations.

3 Construction of the Standard Model.
   - Quantum Electrodynamics (QED).
   - Scattering theory - Calculation of a squared matrix element.
   - Weak interactions.
   - The electroweak theory.
   - Quantum Chromodynamics.

4 Beyond the Standard Model of particle physics.
   - The Standard Model: advantages and open questions.
   - Grand unified theories.
   - Supersymmetry.
   - Extra-dimensional theories.
   - String theory.

5 Summary.
Building blocks describing matter.

- **Atom compositness.**
  - Neutrons.
  - Protons.
  - **Electrons.**

- **Proton and neutron compositness.**
  - Naively: up and down quarks.
  - In reality: dynamical objects made of
    - Valence and sea quarks.
    - Gluons [see below...].

- **Beta decays.**
  - \( n \rightarrow p + e^- + \bar{\nu}_e. \)
  - Needs for a **neutrino.**
Three families of fermionic particles [Why three?]

- **Quarks:**

<table>
<thead>
<tr>
<th>Family</th>
<th>Up-type quark</th>
<th>Down-type quark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} generation</td>
<td>up quark ( u )</td>
<td>down quark ( d )</td>
</tr>
<tr>
<td>2\textsuperscript{nd} generation</td>
<td>charm quark ( c )</td>
<td>strange quark ( s )</td>
</tr>
<tr>
<td>3\textsuperscript{rd} generation</td>
<td>top quark ( t )</td>
<td>bottom quark ( b )</td>
</tr>
</tbody>
</table>

- **Leptons:**

<table>
<thead>
<tr>
<th>Family</th>
<th>Charged lepton</th>
<th>Neutrino</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} generation</td>
<td>electron ( e^- )</td>
<td>electron neutrino ( \nu_e )</td>
</tr>
<tr>
<td>2\textsuperscript{nd} generation</td>
<td>muon ( \mu^- )</td>
<td>muon neutrino ( \nu_\mu )</td>
</tr>
<tr>
<td>3\textsuperscript{rd} generation</td>
<td>tau ( \tau^- )</td>
<td>tau neutrino ( \nu_\tau )</td>
</tr>
</tbody>
</table>

- In addition, the associated antiparticles.
- The only difference between generations lies in the (increasing) mass.

**Experimental status** [Particle Data Group Review].

* All these particles have been observed.
Fundamental interactions and gauge bosons.

- **Electromagnetism.**
  - Interactions between charged particles (quarks and charged leptons).
  - Mediated by **massless photons** $\gamma$ (spin one).

- **Weak interaction.**
  - Interactions between the left-handed components of the fermions.
  - Mediated by **massive weak bosons** $W^{\pm}$ and $Z^0$ (spin one).
  - **Self interactions** between $W^{\pm}$ and $Z^0$ bosons (and photons) [see below...].

- **Strong interactions.**
  - Interactions between **colored particles** (quarks).
  - Mediated by **massless gluons** $g$ (spin one).
  - **Self interactions** between gluons [see below...].
  - Hadrons and mesons are made of quarks and gluons.
  - At the nucleus level: binding of protons and neutrons.

- **Gravity.**
  - Interactions between all particles.
  - Mediated by the **(non-observed) massless graviton** (spin two).
  - **Not described by the Standard Model.**
  - Attempts: superstrings, $M$-theory, quantum loop gravity, ...
The Standard Model of particle physics - framework (1).

**Symmetry principles ↔ elementary particles and their interactions.**

* Compatible with **special relativity**.
  - Minkowski spacetime with the metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.
  - Scalar product $x \cdot y = x^\mu y_\mu = x^\mu y^\nu \eta_{\mu\nu} = x^0 y^0 - \vec{x} \cdot \vec{y}$.
  - Invariance of the speed of light $c$.
  - Physics independent of the inertial reference frame.

* Compatible with **quantum mechanics**.
  - Classical fields: relativistic analogous of wave functions.

* **Quantum field theory**.
  - Quantization of the fields: harmonic and fermionic oscillators.

* Based on **gauge theories** [see below...].

**Conventions.**

* $\hbar = c = 1$ and $\eta_{\mu\nu}, \eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

* Raising and lowering indices: $V^\mu = \eta^{\mu\nu} V_\nu$ and $V_\mu = \eta_{\mu\nu} V^\nu$.

* Indices.
  - Greek letters: $\mu, \nu \ldots = 0, 1, 2, 3$.
  - Roman letters: $i, j, \ldots = 1, 2, 3$. 
What is a symmetry?

- A symmetry operation leaves the laws of physics invariant.  
  e.g., Newton’s law is the same in any inertial frame: \( \vec{F} = m \frac{d^2 \vec{x}}{dt^2} \).

Examples of symmetry.

- Spacetime symmetries: rotations, Lorentz boosts, translations.
- Internal symmetries: quantum mechanics: \( |\psi\rangle \rightarrow e^{i\alpha} |\psi\rangle \).

Nøther theorem.

- To each symmetry is associated a conserved charge.
- Examples: electric charge, energy, angular momentum, ...
Dynamics is based on symmetry principles.

* **Spacetime symmetries (Poincaré).**
  Particle types: scalars, spinors, vectors, ...
  **Beyond:** supersymmetry, extra-dimensions.

* **Internal symmetries (gauge interactions).**
  Electromagnetism, weak and strong interactions.
  **Beyond:** Grand Unified Theories.

**Importance of symmetry breaking and anomalies** [see below...].

* Masses of the **gauge bosons**.
* Generation of the **fermion masses**.
* **Quantum numbers** of the particles.
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5 Summary.
Euler-Lagrange equations - theoretical concepts.

- **We consider a set of fields** \( \phi(x^\mu) \).
  - They depend on **spacetime** coordinates (relativistic).

- **A system is described by a Lagrangian** \( \mathcal{L}(\phi, \partial_\mu \phi) \) where \( \partial_\mu \phi = \frac{\partial \phi}{\partial x^\mu} \).
  - **Variables**: the fields \( \phi \) and their first-order derivatives \( \partial_\mu \phi \).

- **Action**.
  - Related to the Lagrangian \( S = \int d^4x \mathcal{L} \).

- **Equations of motion**.
  - Dynamics described by the **principle of least action**.
  - Leads to **Euler-Lagrange** equations:
    \[
    \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} = 0 \quad \text{where} \ \phi \text{ and } \partial_\mu \phi \text{ are taken independent.}
    \]
Euler-Lagrange equations - example.

- **The electromagnetic potential** \( A^\mu(x) = (V(t, \vec{x}), \vec{A}(t, \vec{x})) \).

- **External electromagnetic current:** \( j^\mu(x) = (\rho(t, \vec{x}), \vec{j}(t, \vec{x})) \).

- **The system is described by the Lagrangian** \( \mathcal{L} \) (the action \( S = \int d^4x \mathcal{L} \)).

\[
\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu j^\mu \quad \text{with} \quad F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix}
0 & -E^1 & -E^2 & -E^3 \\
E^1 & 0 & -B^3 & B^2 \\
E^2 & B^3 & 0 & -B^1 \\
E^3 & -B^2 & B^1 & 0
\end{pmatrix}.
\]

[Einstein conventions: repeated indices are summed.]

- **Equations of motion.**
  - The **Euler-Lagrange** equations are
  \[
  \frac{\partial \mathcal{L}}{\partial A_\mu} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\nu)} = 0 \sim \partial_\mu F^{\mu\nu} = j_\nu \sim \begin{cases}
  \vec{\nabla} \cdot \vec{E} = \rho \\
  \vec{\nabla} \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t}
\end{cases}.
  \]

  - The **constraint equations** come from
  \[
  \partial_\mu F^{\nu\rho} + \partial_\nu F^{\rho\mu} + \partial_\rho F^{\mu\nu} = 0 \sim \begin{cases}
  \vec{\nabla} \cdot \vec{B} = 0 \\
  \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\end{cases}.
  \]
Symmetries.

- **What is an invariant Lagrangian under a symmetry?**
  - We associate an **operator (or matrix)** $G$ to the symmetry:
    \[
    \phi(x) \rightarrow G\phi(x) \quad \text{and} \quad \mathcal{L} \rightarrow \mathcal{L} + \partial_\mu(\ldots).
    \]
  - The action is thus **invariant**.

- **Symmetries in quantum mechanics.**
  - Wigner: $G$ is a (anti)-unitary operator.
  - For unitary operators, $\exists g$, hermitian, so that
    \[
    G = \exp [ig] = \exp [i\alpha^i g_i].
    \]
  - $\alpha^i$ are the **transformation parameters**.
  - $g_i$ are the **symmetry generators**.
  - **Example**: rotations $R(\vec{\alpha}) = \exp[-i\vec{\alpha} \cdot \vec{J}]$ ($\vec{J} \equiv$ angular momentum).

- **Symmetry group and algebra.**
  - The product of two symmetries is a symmetry $\Rightarrow \{G_i\}$ **form a group**.
  - This implies that $\{g_i\}$ **form an algebra**.
    \[
    [g_i, g_j] \equiv g_ig_j - g_jg_i = if_{ij}^k g_k.
    \]
  - **Rotations**: $[J_i, J_j] = iJ_k$ with $(i, j, k)$ cyclic.
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5. Summary.
The Poincaré group and quantum field theory.

- **Quantum mechanics** is invariant under the Galileo group.
- **Maxwell equations** are invariant under the Poincaré group.

**Consistency principles.**

* **Relativistic quantum mechanics.**
  Relativistic equations (Klein-Gordon, Dirac, Maxwell, ...)

* **Quantum field theory**
  The field are quantized: second quantization.
  (harmonic and fermionic oscillators).
The Poincaré algebra and the particle masses.

**The Poincaré algebra** reads \((\mu, \nu = 0, 1, 2, 3)\)

\[
\begin{align*}
\left[ L^{\mu\nu}, L^{\rho\sigma} \right] &= -i \left( \eta^{\nu\sigma} L^{\rho\mu} - \eta^{\mu\sigma} L^{\rho\nu} + \eta^{\nu\rho} L^{\mu\sigma} - \eta^{\mu\rho} L^{\nu\sigma} \right), \\
\left[ L^{\mu\nu}, P^\rho \right] &= -i \left( \eta^{\nu\rho} P^\mu - \eta^{\mu\rho} P^\nu \right), \\
\left[ P^\mu, P^\nu \right] &= 0,
\end{align*}
\]

where

* \( L_{\mu\nu} = -L_{\nu\mu} \) is **antisymmetric**.
* \( L_{ij} = J^k \equiv \text{rotations}; \ (i, j, k) \) is a cyclic permutation of \((1, 2, 3)\).
* \( L_{0i} = K^i \equiv \text{boosts} \ (i = 1, 2, 3)\).
* \( P_\mu \equiv \text{spacetime translations} \).

**Beware of the adopted conventions (especially in the literature).**

**The particle masses.**

* A Casimir operator is an operator commuting with all generators. \(\sim \) **quantum numbers**.
* The quadratic Casimir \(Q_2\) reads \(Q_2 = P^\mu P_\mu = E^2 - \vec{p} \cdot \vec{p} = m^2\).
* The masses are the **eigenvalues** of the \(Q_2\) operator.
Reminder: the rotation algebra and its representations.

- **The rotation algebra** reads

\[
[J^i, J^j] = i\varepsilon_{ijk} J^k = \begin{cases} 
 iJ_k & \text{with } (i, j, k) \text{ a cyclic permutation of } (1, 2, 3).
\end{cases}
- \begin{cases} 
 -iJ_k & \text{with } (i, j, k) \text{ an anticyclic permutation of } (1, 2, 3).
\end{cases}
\]

- **The operator** \(\vec{J} \cdot \vec{J}\).

  * Defining \(\vec{J} = (J^1, J^2, J^3)\), we have \([\vec{J} \cdot \vec{J}, J^i] = 0\).
  * \(\vec{J} \cdot \vec{J}\) is thus a **Casimir operator** (commuting with all generators).

- **Representations.**

  * A **representation** is characterized by
    \begin{itemize}
    \item Two numbers: \(j \in \frac{1}{2} \mathbb{N}\) and \(m \in \{-j, -j + 1, \ldots, j - 1, j\}\).
    \end{itemize}
  * The \(J^i\) matrices are \((2j + 1) \times (2j + 1)\) **matrices**.
    \begin{itemize}
    \item \(j = 1/2\): Pauli matrices (over two).
    \item \(j = 1\): usual rotation matrices (in three dimensions).
    \end{itemize}
  * A state is represented by a ket \(|j, m\rangle\) such that
    \[
    J_\pm |j, m\rangle = \sqrt{j(j + 1) - m(m \pm 1)} |j, m \pm 1\rangle,
    \]
    \[
    J^3 |j, m\rangle = m |j, m\rangle \quad \text{and} \quad \vec{J} \cdot \vec{J} |j, m\rangle = j(j + 1) |j, m\rangle.
    \]
The Lorentz algebra and the particle spins.

- **The Lorentz algebra** reads
  \[
  [L^{\mu\nu}, L^{\rho\sigma}] = -i \left( \eta^{\nu\sigma} L^{\rho\mu} - \eta^{\mu\sigma} L^{\rho\nu} + \eta^{\nu\rho} L^{\mu\sigma} - \eta^{\mu\rho} L^{\nu\sigma} \right),
  \]
  
  * We define \( N^i = \frac{1}{2} (J^i + iK^i) \) and \( \bar{N}^i = \frac{1}{2} (J^i - iK^i) \).

  One gets \([N^i, N^j] = -iN^k\), \([\bar{N}^i, \bar{N}^j] = -i\bar{N}^k\) and \([N^i, \bar{N}^j] = 0\).

**Definition of the spin.**

\[
\{ N_i \} \oplus \{ \bar{N}_i \} = \mathfrak{sl}(2) \oplus \mathfrak{sl}(2) \sim \mathfrak{so}(3) \oplus \mathfrak{so}(3).
\]

The representations of \( \mathfrak{so}(3) \) are known:

\[
\left\{ \begin{array}{c}
N^i \\
\bar{N}^i
\end{array} \right\} \rightarrow S \quad \Rightarrow \quad J^i = N^i + \bar{N}^i \rightarrow \text{spin} = S + \bar{S}.
\]

- **The particle spins** are the representations of the Lorentz algebra.
  
  * \((0, 0) \equiv \text{scalar fields}\).
  * \((1/2, 0)\) and \((0, 1/2) \equiv \text{left and right spinors}\).
  * \((1/2, 1/2) \equiv \text{vector fields}\).
Representations of the Lorentz algebra (1).

- **The (four-dimensional) vector representation** $(1/2, 1/2)$.
  - Action on **four-vectors** $X^\mu$.
  - **Generators**: a set of $10$ $4 \times 4$ matrices
    \[(J^{\mu\nu})^\rho_\sigma = -i \left( \eta^{\rho\mu} \delta^\nu_\sigma - \eta^{\rho\nu} \delta^\mu_\sigma \right).\]
  - A **finite Lorentz transformation** is given by
    \[\Lambda_{\left(\frac{1}{2}, \frac{1}{2}\right)} = \exp \left[ \frac{i}{2} \omega_{\mu\nu} J^{\mu\nu} \right],\]
    where $\omega_{\mu\nu} \in \mathbb{R}$ are the transformation parameters.
  - **Example 1**: a rotation with $\alpha = \omega_{12} = -\omega_{21}$,
    \[R(\alpha) = \exp \left[ i\alpha J^{12} \right] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\]
  - **Example 2**: a boost of speed $v = -\tanh \varphi$ with $\varphi = \omega_{01} = -\omega_{10}$,
    \[B(\varphi) = \exp \left[ i\varphi J^{01} \right] = \begin{pmatrix} \cosh \varphi & \sinh \varphi & 0 & 0 \\ \sinh \varphi & \cosh \varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta \gamma & 0 & 0 \\ -\beta \gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.\]
Representations of the Lorentz algebra (2).

- **Pauli matrices in four dimensions.**
  - **Conventions:**
    \[
    \sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
    \]
  - **Definitions:**
    \[
    \sigma^{\mu \alpha \dot{\alpha}} = (\sigma^0, \sigma^i)_{\alpha \dot{\alpha}}, \quad \bar{\sigma}^{\dot{\mu} \dot{\alpha} \alpha} = (\sigma^0, -\sigma^i)_{\dot{\alpha} \alpha}
    \]

  with \( \alpha = 1, 2 \) and \( \dot{\alpha} = \dot{1}, \dot{2} \).
  The (un)dotted nature of the indices is related to Dirac spinors [see below...].

**Beware of the position (lower or upper, first or second) of the indices.**
**Beware of the types (undotted or dotted) of the indices.**
Representations of the Lorentz algebra (3).

- **The left-handed Weyl spinor representation** \((1/2, 0)\).
  - Action on **complex left-handed spinors** \(\psi_\alpha (\alpha = 1, 2)\).
  - Generators: a set of 10 \(2 \times 2\) matrices
    \[
    (\sigma^{\mu \nu})_{\alpha \beta} = -\frac{i}{4} \left( \sigma^\mu \sigma^\nu - \sigma^\nu \sigma^\mu \right)_{\alpha \beta}.
    \]
  - A **finite Lorentz transformation** is given by
    \[
    \Lambda_{(1/2,0)} = \exp \left[ \frac{i}{2} \omega_{\mu \nu} \sigma^{\mu \nu} \right].
    \]

- **The right-handed Weyl spinor representation** \((0, 1/2)\).
  - Action on **complex right-handed spinors** \(\bar{\chi}^{\dot{\alpha}} (\dot{\alpha} = \dot{1}, \dot{2})\).
  - Generators: a set of 10 \(2 \times 2\) matrices
    \[
    (\bar{\sigma}^{\mu \nu})^{\dot{\alpha} \dot{\beta}} = -\frac{i}{4} \left( \bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu \right)^{\dot{\alpha} \dot{\beta}}.
    \]
  - A **finite Lorentz transformation** is given by
    \[
    \Lambda_{(0,1/2)} = \exp \left[ \frac{i}{2} \omega_{\mu \nu} \bar{\sigma}^{\mu \nu} \right].
    \]

- **Complex conjugation maps left-handed and right-handed spinors.**
Representations of the Lorentz algebra (4).

- **Lowering and raising spin indices.**
  
  * We can define a **metric acting on spin space** [Beware of the conventions],

  \[
  \varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \text{and} \quad \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.
  \]

  \[
  \varepsilon_{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \text{and} \quad \varepsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.
  \]

  * One has:

  \[
  \psi_\alpha = \varepsilon_{\alpha\beta} \psi_\beta, \quad \psi^\alpha = \varepsilon^{\alpha\beta} \psi_\beta, \quad \bar{\chi}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{\chi}^{\dot{\beta}}, \quad \bar{\chi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\chi}_{\dot{\beta}}.
  \]

- **Beware of the adopted conventions for the position of the indices (we are summing on the second index).**

  \[
  \varepsilon^{\alpha\beta} \psi_\beta = -\varepsilon^{\beta\alpha} \psi_\beta.
  \]
Dirac matrices in four dimensions (in the chiral representation).

* Definition:
\[ \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}. \]

* The (Clifford) algebra satisfied by the \( \gamma \)-matrices reads
\[ \{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 \eta^{\mu\nu}. \]

* The chirality matrix, i.e., the fifth Dirac matrix is defined by
\[ \gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \{\gamma^5, \gamma^\mu\} = 0. \]
A Dirac spinor is defined as

\[ \psi_D = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \]

which is a reducible representation of the Lorentz algebra.

- Generators of the Lorentz algebra: a set of 10 $4 \times 4$ matrices

\[ \gamma^{\mu\nu} = -\frac{i}{4} [\gamma^\mu, \gamma^\nu] = \begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar{\sigma}^{\mu\nu} \end{pmatrix} \]

- A finite Lorentz transformation is given by

\[ \Lambda_{(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})} = \exp \left[ \frac{i}{2} \omega_{\mu\nu} \gamma^{\mu\nu} \right] = \begin{pmatrix} \Lambda_{(\frac{1}{2}, 0)} & 0 \\ 0 & \Lambda_{(0, \frac{1}{2})} \end{pmatrix}. \]

A Majorana spinor is defined as

\[ \psi_M = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \]

\[ \Leftrightarrow \text{a Dirac spinor with conjugate left- and right-handed components.} \]

\[ \bar{\psi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\beta} \bar{\psi}^{\beta} \quad \text{with} \quad \bar{\psi}^{\beta} = (\psi_\beta)^\dagger. \]
Summary - Representations and particles.

Irreducible representations of the Poincaré algebra vs. particles.

- **Scalar particles** *(Higgs boson).*
  * *(0, 0)* representation.

- **Massive Dirac fermions** *(quarks and leptons* after symmetry breaking).*
  * *(1/2, 0) ⊕ (0, 1/2)* representation.
  * The mass term mixes both spinor representations.

- **Massive Majorana fermions** *(not in the Standard Model ⇒ dark matter).*
  * *(1/2, 0) ⊕ (0, 1/2)* representation.
  * A Majorana field is self conjugate (the particle = the antiparticle).
  * The mass term mixes both spinor representations.

- **Massless Weyl fermions** *(fermions* before symmetry breaking).*
  * *(1/2, 0)* or *(0, 1/2)* representation.
  * The conjugate of a left-handed fermion is right-handed.

- **Massless and massive vector particles** *(gauge bosons).*
  * *(1/2, 1/2)* representation.
# Outline

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2. **Special relativity and gauge theories.**
   - Action and symmetries.
   - Poincaré and Lorentz algebras and their representations.
   - Relativistic wave equations.

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5. **Summary.**
Relativistic wave equations: scalar fields.

- **Definition:**
  - (0, 0) representation of the Lorentz algebra.
  - Lorentz transformation of a scalar field $\phi$
    
    $$\phi(x) \rightarrow \phi'(x') = \phi(x).$$

- **Correspondence principle.**
  - $P_\mu \leftrightarrow i\partial_\mu$.
  - Application to the mass-energy relation: the **Klein-Gordon equation**.
    
    $$P^2 = m^2 \leftrightarrow (\Box + m^2)\phi = 0.$$
  - The associated Lagrangian is given by $\mathcal{L}_{KG} = (\partial_\mu \phi)^\dagger (\partial_\mu \phi) - m^2 \phi^\dagger \phi$,
    cf. Euler-Lagrange equations:
    
    $$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} = 0$$
    
    where $\phi$ and $\partial_\mu \phi$ are taken independent.

**Scalar fields in the Standard Model.**

- The only undiscovered particle is a scalar field: **the Higgs boson**.
- Remark: in supersymmetry, we have a lot of scalar fields [see below...].
Relativistic wave equations: vector fields (1).

**Definition:**

* (1/2, 1/2) representation of the Lorentz algebra.

* Lorentz transformation of a vector field $A^\mu$

\[
A^\mu(x) \to A'^\mu(x') = \Lambda_{\frac{1}{2}, \frac{1}{2}}^{\mu \nu} A^\nu(x) .
\]

**Maxwell equations and Lagrangian.**

* The relativistic Maxwell equations are

\[
\partial_\mu F^{\mu \nu} = j^\nu .
\]

* $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor.

* $j^\nu$ is the electromagnetic current.

* The associated Lagrangian is given by

\[
\mathcal{L}_{\text{EM}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - A^\mu j_\mu .
\]

* This corresponds to an Abelian $U(1)$ (commutative) gauge group.
Relativistic wave equations: vector fields (2).

- The non-abelian (non-commutative) group $SU(N)$.
  - The group is of dimension $N^2 - 1$.
  - The algebra is generated by $N^2 - 1$ matrices $T_a$ ($a = 1, \ldots, N^2 - 1$),
    \[ [T_a, T_b] = i f_{ab}^c T_c , \]
    where $f_{ab}^c$ are the structure constants of the algebra.
    Example: $SU(2)$: $f_{ab}^c = \varepsilon_{ab}^c$.
  - Usually employed representations for model building.
    - Fundamental and anti-fundamental: $N \times N$ matrices so that
      \[ \text{Tr}(T_a) = 0 \quad \text{and} \quad T_a^\dagger = T_a . \]
    - Adjoint: $(N^2 - 1) \times (N^2 - 1)$ matrices given by
      \[ (T_a)_b^c = -i f_{ab}^c . \]
  - For a given representation $\mathcal{R}$:
    \[ \text{Tr}(T_a T_b) = \tau_\mathcal{R} \delta_{ab} , \]
    where $\tau_\mathcal{R}$ is the Dynkin index of the representation.
Application to physics.

* We select a **gauge group** (here: $SU(N)$).
* We define a **coupling constant** (here $g$).
* We assign **representations** of the group to matter fields.
* The $N^2 - 1$ gauge bosons are given by $A_\mu = A_\mu^a T_a$.
* The **field strength tensor** is defined by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

$$= \left[ \partial_\mu A_\nu^c - \partial_\nu A_\mu^c + g f_{ab}^c A_\mu^a A_\nu^b \right] T_c .$$

* The associated Lagrangian is given by [Yang, Mills (1954)]

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4\tau R} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) .$$

* Contains **self interactions** of the vector fields.

Vector fields in the Standard Model.

* The gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y$ [see below...].
* The bosons are the photon, the weak $W^\pm$ and $Z^0$ bosons, and the gluons.
Relativistic wave equations: Dirac spinors.

- **Definition:**
  - *(1/2, 0) ⊕ (0, 1/2) representation* of the Poincaré algebra.
  - *Lorentz transformations* of a Dirac field \( \psi_D \)
    \[
    \psi_D(x) \rightarrow \psi_D'(x') = \Lambda(\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \psi_D(x).
    \]

- **Dirac’s idea.**
  - The Klein-Gordon equation is quadratic ⇒ particles and antiparticles.
  - *A conceptual problem in the 1920’s.*
  - Linearization of the d’Alembertian:
    \[
    (i\gamma^\mu \partial_\mu - m)\psi_D = 0 \quad \Leftrightarrow \quad \mathcal{L}_D = \bar{\psi}_D (i\gamma^\mu \partial_\mu - m)\psi_D,
    \]
    where
    - \( \bar{\psi}_D = \psi_D^\dagger \gamma^0.\)
    - \( (\gamma^\mu \partial_\mu)^2 = \Box \Leftrightarrow \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}.\)

---

**Fermionic fields in the Standard Model.**

- Matter ≡ **Dirac spinors after symmetry breaking.**
- Matter ≡ **Weyl spinors before symmetry breaking [see below...].**
Summary - Relativistic wave equations.

- **General properties.**
  - The equations derive from **Poincaré invariance**.

- **Scalar particles** (**Higgs boson**).
  - **Klein-Gordon equation**.

- **Massive Dirac and Majorana fermions** (**quarks and leptons**).
  - **Dirac equation**.

- **Massless and massive vector particles** (**gauge bosons**).
  - **Maxwell equations** (Abelian case).
  - **Yang-Mills equations** (non-Abelian case).
Outline.


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   - Grand unified theories.
   - Supersymmetry.
   - Extra-dimensional theories.
   - String theory.

5. Summary.
Global symmetries for the Dirac Lagrangian.

**Toy model.**
- We select the **gauge group** $SU(N)$ with a coupling constant $g$.
- We assign the **fundamental representations** to the fermion fields $\Psi$,

$$
\Psi = \begin{pmatrix}
\psi_1 \\
\vdots \\
\psi_N
\end{pmatrix}, \quad \bar{\Psi} = (\bar{\psi}_1 \cdots \bar{\psi}_N).
$$

- The **Lagrangian** reads

$$
\mathcal{L} = \bar{\Psi} \left( i \gamma^\mu \partial_\mu - m \right) \Psi.
$$

**The global $SU(N)$ invariance.**
- We define a **global $SU(N)$ transformation** of parameters $\omega^a$,

$$
\Psi(x) \to \Psi'(x) = \exp \left[ + ig \omega^a T^\text{fund}_a \right] \Psi \equiv U \Psi,
\bar{\Psi}(x) \to \bar{\Psi}'(x) = \bar{\Psi} \exp \left[ - ig \omega^a T^\text{fund}_a \right] \equiv \bar{\Psi} U^\dagger.
$$

- **The Lagrangian is invariant,**

$$
\mathcal{L} \to \mathcal{L}.
$$
Gauge symmetries for the Dirac Lagrangian (1).

- **Local (internal) $SU(N)$ invariance.**
  - *Promotion* of the global invariance to a local invariance.
  - We define a local $SU(N)$ transformation of parameters $\omega^a(x)$,
    \[
    \Psi(x) \rightarrow \Psi'(x) = U(x) \Psi, \quad \bar{\Psi}(x) \rightarrow \bar{\Psi}'(x) = \bar{\Psi} U^\dagger(x).
    \]
  - *The Lagrangian is not invariant anymore.*
    \[
    \mathcal{L} = \bar{\Psi} \left( i \gamma^\mu \partial_\mu - m \right) \Psi \quad \not\rightarrow \quad \mathcal{L}.
    \]
  - *Due to:*
    - The spacetime dependence of $U(x)$.
    - The presence of derivatives in the Lagrangian.
  - *Idea:* modification of the derivative.
    - Introduction of a new field with *ad hoc* transformation rules.
    - Recovery of the Lagrangian invariance.
Gauge symmetries for the Dirac Lagrangian (2).

- **Local (internal) $SU(N)$ invariance.**
  - Local invariance is recovered after:
    - The introduction of a new vector field $A^\mu = A^\mu a T^a_{\text{fund}}$ with
      \[
      A^\mu(x) \rightarrow A'^\mu(x) = U(x) \left[ A^\mu(x) + \frac{i}{g} \partial^\mu \right] U^\dagger(x) ,
      \]
      \[
      F^{\mu\nu}(x) \rightarrow U(x) F^{\mu\nu}(x) U^\dagger(x) \Rightarrow \text{Tr}(F^{\mu\nu} F_{\mu\nu}) \rightarrow \text{Tr}(F^{\mu\nu} F_{\mu\nu}) .
      \]
    - The modification of the derivative into a covariant derivative,
      \[
      \partial_\mu \psi(x) \rightarrow D_\mu \psi(x) = \left[ \partial_\mu - ig A_\mu(x) \right] \psi(x) .
      \]
    - **Transformation laws:**
      \[
      D_\mu \psi(x) \rightarrow U(x) D_\mu \psi(x) \Rightarrow \mathcal{L} \rightarrow \mathcal{L} .
      \]
  - This holds (and simplifies) for $U(1)$ gauge invariance. In particular:
    \[
    A^\mu(x) \rightarrow A'^\mu(x) = A^\mu(x) + \partial^\mu \omega(x) .
    \]
  - **Example:** Abelian $U(1)_{\text{e.m.}}$ gauge group for electromagnetism.
Symmetry breaking - theoretical setup.

- **Let us consider a** $U(1)_X$ **gauge symmetry.**
  * Gauge boson $X_\mu$ — gauge coupling constant $g_X$.

- **Matter content.**
  * A set of fermionic particles $\Psi^j$ of charge $q^j_X$.
  * A complex scalar field $\phi$ with charge $q_\phi$.

- **Lagrangian.**
  * Kinetic and gauge interaction terms for all fields.
  
  \[
  \mathcal{L}_{\text{kin}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\Psi}^j i \gamma^\mu D_\mu \Psi^j + (D_\mu \phi)^\dagger (D^\mu \phi) \\
  = -\frac{1}{4} \left( \partial_\mu X_\nu - \partial_\nu X_\mu \right) \left( \partial^\mu X^\nu - \partial^\nu X^\mu \right) \\
  + \bar{\Psi}^j \gamma^\mu \left( i \partial_\mu + g_X q^j_X X_\mu \right) \Psi^j + \left( \partial_\mu + ig_X q_\phi X_\mu \right) \phi^\dagger \left[ \left( \partial^\mu - ig_X q_\phi X^\mu \right) \phi \right].
  \]

  * A scalar potential ($\mathcal{L}_V = -V_{\text{scal}}$) and Yukawa interactions.

  \[
  V_{\text{scal}} = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \quad \text{with} \quad \lambda > 0 , \quad \mu^2 > 0 ,
  \]

  \[
  \mathcal{L}_{\text{Yuk}} = -y^j \phi \bar{\Psi}^j \Psi^j + \text{h.c.} \quad \text{with} \quad y^j \text{ being the Yukawa coupling.}
  \]
Symmetry breaking - minimization of the scalar potential.

- The system lies at the **minimum** of the potential.
  \[
  \frac{dV_{\text{scal}}}{d\phi} = 0 \iff \langle \phi \rangle = \frac{1}{\sqrt{2}} \sqrt{\frac{\mu^2}{\lambda}} e^{i\alpha_0}.
  \]

- \( v = \sqrt{2} \langle \phi \rangle \) is the vacuum expectation value (vev) of the field \( \phi \).

- We define \( \phi \) such that \( \alpha_0 = 0 \).

- We **shift** the scalar field by its vev
  \[
  \phi = \frac{1}{\sqrt{2}} \left[ v + A + i B \right],
  \]
  where \( A \) and \( B \) are **real** scalar fields.

\[
V_{\text{scal}} = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2.
\]
Symmetry breaking - mass eigenstates (1).

We shift the scalar field by its vev.

\[ \phi = \frac{1}{\sqrt{2}} [v + A + iB] . \]

- **Scalar mass eigenstates.**
  - The scalar potential reads now
    \[
    V_{\text{scal}} = \lambda v^2 A^2 + \lambda \left[ \frac{1}{4} A^4 + \frac{1}{4} B^4 + \frac{1}{2} A^2 B^2 + vA^3 + vAB^2 \right] .
    \]
  - One gets **self interactions** between \(A\) and \(B\).
  - \(A\) is a **massive** real scalar field, \(m_A^2 = 2\mu^2\), the so-called **Higgs boson**.
  - \(B\) is a **massless** pseudoscalar field, the so-called **Goldstone boson**.
Symmetry breaking - mass eigenstates (2).

We shift the scalar field by its vev.

\[ \phi = \frac{1}{\sqrt{2}} \left[ v + A + i B \right]. \]

**Gauge boson mass** \( m_X \).

* The kinetic and gauge interaction terms for the scalar field \( \phi \) read now

\[
\left( D^\mu \phi^\dagger \right) \left( D_\mu \phi \right) = \left[ \left( \partial_\mu + ig_X q_\phi X_\mu \right) \phi^\dagger \right] \left[ \left( \partial^\mu - ig_X q_\phi X^\mu \right) \phi \right]
\]

\[
= \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu B \partial^\mu B + \frac{1}{2} g_X^2 v^2 X_\mu X^\mu + \ldots
\]

* One gets **kinetic terms** for the \( A \) and \( B \) fields.

* The dots stand for **bilinear and trilinear interactions** of \( A \), \( B \) and \( X_\mu \).

* The gauge boson becomes massive, \( m_X = g_X v \).

* The Goldstone boson is eaten \( \equiv \) the third polarization state of \( X_\mu \).

* **The gauge symmetry is spontaneously broken.**
Symmetry breaking - mass eigenstates (3).

We shift the scalar field by its vev.

$$\phi = \frac{1}{\sqrt{2}} \left[ v + A + i B \right].$$

- **Fermion masses** $m_j$.
  
  * The Yukawa interactions read now
  
  $$\mathcal{L}_\text{Yuk} = -y_j \phi \bar{\psi}_j \psi^j \rightarrow \frac{1}{\sqrt{2}} y_j v \bar{\psi}_j \psi^j + \frac{1}{\sqrt{2}} y_j (A + i B) \bar{\psi}_j \psi^j.$$  

  * One gets **Yukawa interactions** between $A$, $B$ and $\psi^j$.
  
  * The fermion fields become massive, $m_j = y_j v$. 

Summary - Nœther procedure.

Nœther procedure to get gauge invariant Lagrangians.

1. Choose a **gauge group**.
2. Setup the **matter field content** in a given representation.
3. Start from the **free Lagrangian for matter fields**.
4. Promote derivatives to **covariant derivatives**.
5. Add **kinetic terms** for the gauge bosons ($L_{YM}$ or $L_M$).

**Some remarks:**

- The Nœther procedure holds for both **fermion and scalar** fields.
- This implies that the **interactions are dictated by the geometry**.
- The gauge group and matter content are **not predicted**.
- The symmetry can be eventually **broken**.
- The theory **must be anomaly-free**.
- This holds in **any number of spacetime dimensions**.
- This can be generalized to **superfields** (supersymmetry, supergravity).
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5. Summary.
Theoretical setup.

- **The electromagnetism is the simplest gauge theory.**

- **We consider an Abelian gauge group, \( U(1)_{e.m.} \).**
  * Gauge boson: the photon \( A_\mu \).
  * Gauge coupling constant: the electromagnetic coupling constant \( e \).

- We relate \( e \) to \( \alpha = \frac{e^2}{4\pi} \).

- **Both quantities depend on the energy** (cf. renormalization):
  \[
  \alpha(0) \approx \frac{1}{137} \quad \text{and} \quad \alpha(100\text{GeV}) \approx \frac{1}{128}.
  \]

- **Matter content.**

<table>
<thead>
<tr>
<th>Name</th>
<th>1(^{\text{st}}) gen.</th>
<th>2(^{\text{nd}}) gen.</th>
<th>3(^{\text{rd}}) gen.</th>
<th>Electric charge ( q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charged lepton</td>
<td>( \Psi_e )</td>
<td>( \Psi_\mu )</td>
<td>( \Psi_\tau )</td>
<td>( -1 )</td>
</tr>
<tr>
<td>Neutrino</td>
<td>( \Psi_{\nu_e} )</td>
<td>( \Psi_{\nu_\mu} )</td>
<td>( \Psi_{\nu_\tau} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>Up-type quarks</td>
<td>( \Psi_u )</td>
<td>( \Psi_c )</td>
<td>( \Psi_t )</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>Down-type quarks</td>
<td>( \Psi_d )</td>
<td>( \Psi_s )</td>
<td>( \Psi_b )</td>
<td>( -\frac{1}{3} )</td>
</tr>
</tbody>
</table>
Lagrangian.

- We start from the free Lagrangian,

\[ \mathcal{L}_{\text{free}} = \sum_{j=e,\nu_e,u,d,...} \bar{\Psi}_j i\gamma^\mu \partial_\mu \Psi^j. \]

- The Nœther procedure leads to

\[ \mathcal{L}_{\text{QED}} = \sum_{j=e,\nu_e,u,d,...} \bar{\Psi}_j i\gamma^\mu D_\mu \Psi^j - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

with \( D_\mu = \partial_\mu - ieq A_\mu \), \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \).

- The electromagnetic interactions are given by

\[ \mathcal{L}_{\text{int}} = \sum_{j=e,u,d,...} \bar{\Psi}_j eq\gamma^\mu A_\mu \Psi^j. \]

≡ photon-fermion-antifermion vertices:

* \( \gamma^\mu \) \( \sim \) the fermions couple through their spin.

* \( q \) \( \sim \) the fermions couple through their electric charge.
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5. Summary.
From Lagrangians to practical computations (1).

- **Scattering theory.**
  * Initial state $i(t)$ at a date $t$.
  * Evolution to a date $t'$.
  * Transition to a final state $f(t')$ (at the date $t'$).
  * The transition is related to the so-called $S$-matrix:
    \[
    S_{fi} = \langle f(t') | i(t') \rangle = \langle f(t') | S | i(t) \rangle.
    \]

- **Perturbative calculation of $S_{fi}$.**
  * $S_{fi}$ is related to the path integral
    \[
    \int d(\text{fields}) \ e^{i \int d^4x L(x)},
    \]
  * $S_{fi}$ can be perturbatively expanded as:
    \[
    S_{fi} = \delta_{fi} + i \left[ \int d^4x L(x) \right]_{fi} - \frac{1}{2} \left[ \int d^4x d^4x' T \{ L(x) L(x') \} \right]_{fi} + \ldots
    \]
    \[
    = \text{no interaction} + \text{one interaction} + \text{two interactions} + \ldots
    \]
    \[
    = \delta_{fi} + iT_{fi}.
    \]

- We need to calculate $T_{fi}$. 

Example in QED with one interaction: the $e^+e^- \rightarrow \gamma$ process.

* The Lagrangian is given by

$$\mathcal{L}_{\text{QED}} = \sum_{j=e,\nu_e,u,d,\ldots} \bar{\Psi}_j i\gamma^\mu D_\mu \Psi^j - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

with

$$D_\mu = \partial_\mu - ie_q A_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

* Initial state $i = e^+e^-$ and final state: $f = \gamma$.

* One single interaction term containing the $\Psi_e, \bar{\Psi}_e$ and $A_\mu$ fields.

$$\mathcal{L}_{\text{QED}} \Rightarrow -e \bar{\Psi}_e \gamma^\mu A_\mu \Psi^e.$$

* The corresponding contribution to $S_{fi}$ reads

$$i \left[ \int d^4x \mathcal{L}(x) \right]_{fi} = i \int d^4x \left[ -e \bar{\Psi}_e \gamma^\mu A_\mu \Psi^e \right].$$

More than one interaction.

* Intermediate, virtual particles are allowed.
  e.g.: $e^+e^- \rightarrow \mu^+\mu^- \rightarrow e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-.$

* Same principles, but accounting in addition for chronology.
From Lagrangians to practical computations (3).

- We consider the specific process \( i_1(p_a) + i_2(p_b) \rightarrow f_1(p_1) + \ldots + f_n(p_n) \).
  
  * The initial state is \( i(t) = i_1(p_a), i_2(p_b) \) (as in colliders).
  
  * The \( n \)-particle final state is \( f(t') = f_1(p_1), \ldots, f_n(p_n) \).

  * \( p_a, p_b, p_1, \ldots, \) and \( p_n \) are the four-momenta.

- We solve the equations of motion and the fields are expanded as plane waves.

\[
\psi = \int d^4 p \left[ (\ldots) e^{-ip\cdot x} + (\ldots) e^{+ip\cdot x} \right] \ldots
\]

  * The unspecified terms correspond to \textit{annihilation/creation operators of (anti)particles} (harmonic and fermionic oscillators).

- We inject these solutions in the Lagrangian.

  * Integrating the exponentials leads to \textit{momentum conservation}.

\[
\int d^4 x \left[ e^{-ip_a\cdot x} e^{-ip_b\cdot x} \prod_j e^{-ip_j\cdot x} \right] = (2\pi)^4 \delta^{(4)}(p_a + p_b - \sum_j p_j).
\]
From Lagrangians to practical computations (4).

- We define the matrix element.

\[ iT_{fi} = (2\pi)^4 \delta^{(4)}(p_a + p_b - \sum_j p_j) i M_{fi} . \]

- By definition, the total cross section:
  * Is the **total production rate** of the final state from the initial state.
  * Requires an **integration over all final state configurations**.
  * Requires an **average over all initial state configurations**.

\[ \sigma = \frac{1}{F} \int dP S^{(n)} |M_{fi}|^2 . \]

- The differential cross section with respect to a kinematical variable \( \omega \) is

\[ \frac{d\sigma}{d\omega} = \frac{1}{F} \int dP S^{(n)} |M_{fi}|^2 \delta\left(\omega - \omega(p_a, p_b, p_1, \ldots, p_n)\right) . \]
From Lagrangians to practical computations (5).

\[ \sigma = \frac{1}{F} \int d\text{PS}^{(n)} |M_{fi}|^2. \]

- The integration over phase space (cf. final state) reads

\[ \int d\text{PS}^{(n)} = \int (2\pi)^4 \delta^{(4)}(p_a + p_b - \sum_j p_j) \prod_j \left[ \frac{d^4p_j}{(2\pi)^4} \delta(p_j^2 - m_j^2) \theta(p_j^0) \right]. \]

  * It includes momentum conservation.
  * It includes mass-shell conditions.
  * The energy is positive.
  * We integrate over all final state momentum configurations.

- The flux factor \( F \) (cf. initial state) reads

\[ \frac{1}{F} = \frac{1}{4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}}. \]

  * It normalizes \( \sigma \) with respect to the initial state density by surface unit.
The squared matrix element $|M_{fi}|^2$

* Is **averaged** over the initial state quantum numbers and spins.
* Is **summed** over the final state quantum numbers and spins.
* Can be calculated with the **Feynman rules** derived from the Lagrangian.
  ◦ **External particles**: spinors, polarization vectors, . . . .
  ◦ **Intermediate particles**: propagators.
  ◦ **Interaction vertices**.

**External particles.**
* Rules derived from the **solutions of the equations of motion**.

**Propagators.**
* Rules derived from the **free Lagrangians**.

**Vertices.**
* Rules directly extracted from the **interaction terms** of the Lagrangian.
From Lagrangians to practical computations (7).

- Feynman rules for external particles (spinors, polarization vectors).

\[ \psi = \int d^4 p \left[ (\ldots) e^{-ip \cdot x} + (\ldots) e^{+ip \cdot x} \right] \ldots \]

* Obtained after solving Dirac and Maxwell equations.

* They are the physical degrees of freedom (included in the dots).

* We do not need their explicit forms for practical calculations [see below...].
Interactions and propagators.

QED Lagrangian.

\[ \mathcal{L}_{\text{QED}} = \sum_{j=e,\nu,e,u,d,...} \bar{\Psi}_j i\gamma^\mu \left( \partial_\mu - ieqA_\mu \right) \Psi_j - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \]

\[ \bar{\Psi}(p_1) \quad \xrightarrow{\text{ieq}\gamma^\mu} \quad A_\mu \quad i\gamma^\mu \quad \Psi(p) \quad \xrightarrow{i\frac{\gamma^\mu p_\mu + m}{p^2 - m^2}} \quad \bar{\Psi}(p_2) \]

- We need to fix the gauge to derive the photon propagator.
- Feynman gauge: \( \partial_\mu A^\mu = 0 \).
- Any other theory would lead to similar rules.
Example of a calculation: $e^+ e^- \rightarrow \mu^+ \mu^- \ (1)$.

- Drawing of the Feynman diagram, using the available Feynman rules.

\[
\begin{align*}
\nonumber e^-(p_b) & \quad \text{	extbullet} \quad \mu^-(p_2) \\
\nonumber e^+(p_a) & \quad \text{	extbullet} \quad \mu^+(p_1) \\
\nonumber A & \quad \text{	extbullet} \quad A
\end{align*}
\]

- Amplitude $iM$ from the Feynman rules (following reversely the fermion lines).

\[
iM = \left[ \bar{v}_{s_a}(p_a) \left( -ie\gamma^\mu \right) u_{s_b}(p_b) \right] \left[ \bar{u}_{s_2}(p_2) \left( -ie\gamma^\nu \right) v_{s_1}(p_1) \right] \frac{-i\eta_{\mu\nu}}{(p_a + p_b)^2}.
\]
Example of a calculation: \( e^+ e^- \rightarrow \mu^+ \mu^- \) (2).

- Derivation of the conjugate amplitude \(-iM^\dagger\).

\[
iM = \left[ \bar{v}_{s_a}(p_a) (-ie\gamma^\mu) u_{s_b}(p_b) \right] \left[ \bar{u}_{s_2}(p_2) (-ie\gamma^\nu) v_{s_1}(p_1) \right] \frac{-i\eta_{\mu\nu}}{(p_a + p_b)^2},
\]

\[-iM^\dagger = \left[ \bar{u}_{s_b}(p_b) (ie\gamma^\mu) v_{s_a}(p_a) \right] \left[ \bar{v}_{s_1}(p_1) (ie\gamma^\nu) u_{s_2}(p_2) \right] \frac{i\eta_{\mu\nu}}{(p_a + p_b)^2}.\]

* Definitions: \( \bar{u} = u^\dagger \gamma^0 \) and \( \bar{v} = v^\dagger \gamma^0 \).
* We remind that \( (\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0 \).
* We remind that \( \gamma^0 \gamma^0 = 1 \) and \( (\gamma^0)^\dagger = \gamma^0 \).

- Computation of the squared matrix element \( |M|^2 \).

\[
|M|^2 = \frac{1}{2} \frac{1}{2} (iM) (-iM^\dagger). 
\]

* We average over the initial electron spin \( \sim 1/2 \).
* We average over the initial positron spin \( \sim 1/2 \).
Example of a calculation: \( e^+ e^- \rightarrow \mu^+ \mu^- \) (3).

- **Computation of the squared matrix element** \( |M|^2 \).

\[
|M|^2 = \frac{e^4}{4(p_a + p_b)^4} \text{Tr} \left[ \gamma^\mu (\not{p}_b + m_e) \gamma^\rho (\not{p}_a - m_e) \right] \text{Tr} \left[ \gamma_\mu (\not{p}_1 - m_\mu) \gamma_\rho (\not{p}_2 + m_\mu) \right].
\]

* We have performed a **sum over all the particle spins**.

* We have introduced \( \not{p} = \gamma^\nu p_\nu \), the electron and muon masses \( m_e \) and \( m_\mu \).

* We have used the properties derived from the Dirac equation

\[
\sum_s u_s(p) \bar{u}_s(p) = \not{p} + m \quad \text{and} \quad \sum_s v_s(p) \bar{v}_s(p) = \not{p} - m.
\]

* For completeness, Maxwell equations tell us that

\[
\sum_\lambda \epsilon^{\mu}_\lambda(p) \epsilon^\nu_\lambda (p) = -\eta^{\mu \nu}. \quad \text{[This relation is gauge-dependent.]}\]
Example of a calculation: $e^+ e^- \rightarrow \mu^+ \mu^-$ (4).

- Simplification of the traces, in the massless case.

\[
|M|^2 = \frac{8e^4}{(p_a + p_b)^4} \left[ (p_b \cdot p_1)(p_a \cdot p_2) + (p_b \cdot p_2)(p_a \cdot p_1) \right].
\]

* We have used the properties of the Dirac matrices

\[
\begin{align*}
\text{Tr} \left[ \gamma^{\mu_1} \ldots \gamma^{\mu_{2k+1}} \right] &= 0, \\
\text{Tr} \left[ \gamma^\mu \gamma^\nu \right] &= 4\eta^{\mu\nu}, \\
\text{Tr} \left[ \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \right] &= 4 \left( \eta^{\mu\nu} \eta^{\rho\sigma} - \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} \right), \\
\text{Tr} \left[ \gamma^5 \right] &= 0, \\
\text{Tr} \left[ \gamma^5 \gamma^\mu \gamma^\nu \right] &= 0, \\
\text{Tr} \left[ \gamma^5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \right] &= 4i\epsilon^{\mu\nu\rho\sigma} \quad \text{with} \quad \epsilon_{0123} = 1.
\end{align*}
\]
Example of a calculation: \( e^+ e^- \rightarrow \mu^+ \mu^- \) (5).

- **Mandelstam variables and differential cross section.**

\[
|M|^2 = \frac{2e^4}{s^2} [t^2 + u^2] \Rightarrow \frac{d\sigma}{dt} = \frac{e^4}{8\pi s^4} [t^2 + u^2] .
\]

* We have introduced the Mandelstam variables

\[
s = (p_a + p_b)^2 = (p_1 + p_2)^2 ,
\]

\[
t = (p_a - p_1)^2 = (p_b - p_2)^2 ,
\]

\[
u = (p_a - p_2)^2 = (p_b - p_1)^2 .
\]

- **Remark: sub-processes names according to the propagator.**

\[\text{s-channel} \quad \text{t-channel} \quad \text{u-channel}\]
## Calculation of a matrix element.

1. **Extraction of the Feynman rules** from the Lagrangian.
2. **Drawing of all possible Feynman diagrams** for the considered process.
3. **Derivation of the transition amplitudes** using the Feynman rules.
4. **Calculation of the squared matrix element.**
   - Sum/average over **finalitial internal quantum numbers**.
   - Calculation of **traces of Dirac matrices**.
   - Possible use of the **Mandelstam variables**.
Outline.

1 Context.

2 Special relativity and gauge theories.
   - Action and symmetries.
   - Poincaré and Lorentz algebras and their representations.
   - Relativistic wave equations.

3 Construction of the Standard Model.
   - Quantum Electrodynamics (QED).
   - Scattering theory - Calculation of a squared matrix element.
   - Weak interactions.
     - The electroweak theory.
     - Quantum Chromodynamics.

4 Beyond the Standard Model of particle physics.
   - The Standard Model: advantages and open questions.
   - Grand unified theories.
   - Supersymmetry.
   - Extra-dimensional theories.
   - String theory.

5 Summary.
The Fermi model of weak interactions (1).

- **Proton decay (Hahn and Meitner, 1911).**

\[ p \rightarrow n + e^+ . \]

* Momentum conservation **fixes final state energies to a single value** (depending on the proton energy).
* Observation: the energy spectrum of the electron is continuous.

- **Solution (Pauli, 1930): introduction of the neutrino.**

\[ p \rightarrow n + e^+ + \nu_e \iff u \rightarrow d + e^+ + \nu_e \] at the quark level.

* Reminder: \( p = uud \) (naively).
* Reminder: \( n = udd \) (naively).
* \( 1 \rightarrow 3 \) particle process: **continuous electron energy spectrum.**

- **How to construct a Lagrangian describing beta decays?**
The Fermi model of weak interactions (2).

- Phenomenological model based on four-point interactions (Fermi, 1932).

\[ \mathcal{L}_{\text{Fermi}} = -2\sqrt{2}G_F \left[ \bar{\Psi}_d \gamma_\mu \frac{1 - \gamma^5}{2} \Psi_u \right] \left[ \bar{\Psi}_{\nu_e} \gamma_\mu \frac{1 - \gamma^5}{2} \Psi_e \right] + \text{h.c.} \]

- Phenomenological model \( \iff \) reproducing experimental data.
- Based on four-fermion interactions.
- The coupling constant \( G_F \) is measured.
- \( G_F = 1.163710^{-5} \, \text{GeV}^{-2} \) is dimensionful.
The Fermi model of weak interactions (3).

- **The Fermi Lagrangian can be rewritten as**

\[
\mathcal{L}_{\text{Fermi}} = -2\sqrt{2} G_F \left[ \bar{\Psi}_d \gamma_\mu \frac{1 - \gamma^5}{2} \Psi_u \right] \left[ \bar{\Psi}_{\nu e} \gamma_\mu \frac{1 - \gamma^5}{2} \Psi_e \right] + \text{h.c.}
\]

\[
= -2\sqrt{2} G_F H_\mu L^\mu + \text{h.c.}
\]

* It contains a **leptonic piece** \( L^\mu \) and a **quark piece** \( H_\mu \).
* Both pieces have the **same structure**.

- **The structure of the weak interactions**
  * The leptonic piece \( L^\mu \) has a **V − A** structure:

\[
L^\mu = \bar{\Psi}_{\nu e} \gamma_\mu \frac{1 - \gamma^5}{2} \Psi_e = \frac{1}{2} \bar{\Psi}_{\nu e} \gamma_\mu \Psi_e - \frac{1}{2} \bar{\Psi}_{\nu e} \gamma_\mu \gamma^5 \Psi_e
\]

* Similarly, the quark piece \( H_\mu \) has a **V − A** structure.
* **The Fermi Lagrangian contains thus** \( VV, AA \) and \( VA \) terms.

- **Behavior under parity transformations.**
  * Under a parity transformation: \( V \rightarrow -V \) and \( A \rightarrow A \).
  * The **VA terms** (and thus weak interactions) violate parity.
  * Parity violation has been observed experimentally (Wu et al., 1956).
The Fermi model of weak interactions (4).

**Analysis of the currents** $L^\mu$ and $H^\mu$.

\[
L^\mu = \bar{\psi}_\nu \gamma^\mu \frac{1 - \gamma^5}{2} \psi_e \quad \text{and} \quad H^\mu = \bar{\psi}_d \gamma^\mu \frac{1 - \gamma^5}{2} \psi_u.
\]

* Presence of the **left-handed chirality projector** $P_L = (1 - \gamma^5)/2$.

**Projectors and their properties.**

* The **chirality projectors** are given by

\[
P_L = \frac{1 - \gamma^5}{2} \quad \text{and} \quad P_R = \frac{1 + \gamma^5}{2}.
\]

* They fulfill the **properties**

\[
P_L + P_R = 1, \quad P_L^2 = P_L \quad \text{and} \quad P_R^2 = P_R.
\]

* If $\Psi$ is a Dirac spinor, **left and right** associated spinors are recovered by

\[
\Psi_D = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad \Psi_L = P_L \Psi_D = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \Psi_R = P_R \Psi_D = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}.
\]

* Only left-handed fermions are sensitive to the weak interactions.
The Fermi model of weak interactions (5).

- **Introducing the left-handed chirality projector** $P_L = 1/2(1 - \gamma^5)$:

  $$L^\mu = \bar{\Psi}_{\nu e}\gamma^\mu P_L \Psi_e = \bar{\Psi}_{\nu e, L}\gamma^\mu \Psi_{e, L} \quad \text{and} \quad (L^\mu)^\dagger = \bar{\Psi}_{\nu e}\gamma^\mu P_L \Psi_{\nu e} = \bar{\Psi}_{\nu e, L}\gamma^\mu \Psi_{\nu e, L} ,$$

  $$H^\mu = \bar{\Psi}_d\gamma^\mu P_L \Psi_u = \bar{\Psi}_{d, L}\gamma^\mu \Psi_{u, L} \quad \text{and} \quad (H^\mu)^\dagger = \bar{\Psi}_u\gamma^\mu P_L \Psi_d = \bar{\Psi}_{u, L}\gamma^\mu \Psi_{d, L} .$$

- **Behavior of the fields under the weak interactions.**
  * Left-handed electron and neutrino behave similarly.
  * Up and down quarks behave similarly.

- **Idea**: group into doublets the left-handed components of the fields:

  $$L_e = \begin{pmatrix} \Psi_{\nu e, L} \\ \Psi_{e, L} \end{pmatrix} \quad \text{and} \quad Q = \begin{pmatrix} \Psi_{u, L} \\ \Psi_{d, L} \end{pmatrix} .$$

- **The currents are then rewritten as:**

  $$L^\mu = \bar{\Psi}_{\nu e, L}\gamma^\mu \Psi_{e, L} = \bar{L}_e\gamma^\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} L_e ,$$

  $$(L^\mu)^\dagger = \bar{\Psi}_{e, L}\gamma^\mu \Psi_{\nu e, L} = \bar{L}_e\gamma^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} L_e .$$

[Similar expressions hold for the quark piece].
From Fermi model to $SU(2)_L$ gauge theory (1).

- **Problems of the Fermi model.**
  
  $$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2}G_F H_\mu L^\mu + \text{h.c.} .$$

  * Issues with quantum corrections, *i.e.*, non-renormalizability.
  * Effective theory valid up to an energy scale $E \ll m_w \approx 100$ GeV.
  * Fermi model is not based on gauge symmetry principles.

- **Solution: a gauge theory (Glashow, Salam, Weinberg, 60-70, [Nobel prize, 1979]).**
  
  * Four fermion interactions can be seen as a $s$-channel diagram.
  * Introduction of a new gauge boson $W_\mu$.
  * This boson couples to fermions with a strength $g_w$.

  $$g^2_w \frac{\gamma^2}{p^2 - m^2_w} \approx -g^2_w \frac{m^2_w}{m^2_w}$$

  * Prediction: $g_w \sim O(1) \Rightarrow m_w \sim 100$ GeV.
From Fermi model to $SU(2)_L$ gauge theory (2).

- **Choice of the gauge group:** suggested by the currents:

  \[
  L^\mu = \bar{L}_e \gamma^\mu \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \Leftrightarrow \bar{L}_e \gamma^\mu \frac{\sigma^1 + i \sigma^2}{2} L_e,
  \]

  \[
  (L^\mu)^\dagger = \bar{L}_e \gamma^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \Leftrightarrow \bar{L}_e \gamma^\mu \frac{\sigma^1 - i \sigma^2}{2} L_e.
  \]

  [Similar expressions hold for the quark piece].

  * Two **Pauli matrices** appear naturally.
  * $\sigma^i/2$ are the generators of the $SU(2)$ algebra
    (in the **fundamental (dimension 2) representation**).

  **We choose the $SU(2)$ gauge group to describe weak interactions.**
We choose the $SU(2)_L$ gauge group to describe weak interactions.

- $1/2\sigma^i$ are the generators of the fundamental representation.
  \[
  \left[ \frac{1}{2} \sigma^i, \frac{1}{2} \sigma^j \right] = i\epsilon^{ij}_k \frac{1}{2} \sigma^k ,
  \]

- The left-handed doublets lie in the fundamental representation $\bf{2}$.
  * The left-handed fields are the only ones sensible to weak interactions.
  * A doublet is a two-dimensional object.
  * The Pauli matrices are $2 \times 2$ matrices.
  * This explains the $L$-subscript in $SU(2)_L$.

- The right-handed leptons lie in the trivial representation $\bf{1}$.
  * Non-sensible to weak interactions.

- $SU(2)_L \sim$ three gauge bosons $W^i_\mu$ with $i = 1, 2, 3$. 
The $SU(2)_L$ gauge theory for weak interactions (1).

- **How to construct the $SU(2)_L$ Lagrangian?**

- **We start from the free Lagrangian for fermions.**
  - *Simplification-1: no quarks here.*
  - *Simplification-2: no right-handed neutrinos.*

\[ \mathcal{L}_{\text{free}} = \bar{L}_e \left( i \gamma^\mu \partial_\mu \right) L_e + \bar{e}_R \left( i \gamma^\mu \partial_\mu \right) e_R . \]

- A mass term mixes left and right-handed fermions.
- The mass term are forbidden since $L_e \sim 2$ and $e_R \sim 1$.

- **We make the Lagrangian invariant under $SU(2)_L$ gauge transformations.**
  - $SU(2)_L$ gauge transformations are given by

\[ L_e \rightarrow \exp \left[ ig_w \omega_i (x) \frac{\sigma^i}{2} \right] L_e = U(x) L_e \quad \text{and} \quad e_R \rightarrow e_R . \]

- Gauge invariance requires covariant derivatives,

\[ \partial_\mu L_e \rightarrow D_\mu L_e = \left[ \partial_\mu - ig_w W_{\mu i} \frac{\sigma^i}{2} \right] L_e \quad \text{and} \quad \partial_\mu e_R \rightarrow D_\mu e_R = \partial_\mu e_R . \]

- We have introduced one gauge boson for each generator ⇒ three $W_{\mu i}$. 
The $SU(2)_L$ gauge theory for weak interactions (2).

- The matter sector Lagrangian reads then.

$$\mathcal{L}_{\text{weak,matter}} = \bar{L}_e \left( i \gamma^\mu D_\mu \right) L_e + \bar{e}_R \left( i \gamma^\mu D_\mu \right) e_R .$$

with

$$D_\mu L_e = \left[ \partial_\mu - ig_w W_{\mu i} \frac{\sigma^i}{2} \right] L_e \quad \text{and} \quad D_\mu e_R = \partial_\mu e_R .$$

- We must then add kinetic terms for the gauge bosons:

$$\mathcal{L}_{\text{weak,gauge}} = -\frac{1}{4} W_{\mu \nu}^i W_i^{\mu \nu} .$$

* The field strength tensor reads:

$$W_{\mu \nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g_w \epsilon_{ijk} W_\mu^j W_\nu^k .$$

* Gauge invariance implies the transformation laws:

$$\frac{\sigma^i}{2} W_i^\mu \to U \left[ \frac{\sigma^i}{2} W_i^\mu + \frac{i}{g_w} \partial^\mu \right] U^\dagger .$$
The \( SU(2)_L \) gauge theory for weak interactions (3).

The weak interaction Lagrangian for leptons.

\[
\mathcal{L}_{\text{weak},e} = \bar{L}_e \left( i \gamma^\mu D_\mu \right) L_e + \bar{e}_R \left( i \gamma^\mu D_\mu \right) e_R - \frac{1}{4} W^{i\mu\nu} W_{i\mu\nu}.
\]

with

\[
D_\mu L_e = \left[ \partial_\mu - ig W_{\mu i} \frac{\sigma^i}{2} \right] L_e,
\]
\[
D_\mu e_R = \partial_\mu e_R,
\]
\[
W^{i\mu\nu} = \partial_\mu W^i_{\nu} - \partial_\nu W^i_{\mu} + g w \epsilon^{ijk} W^j_{\mu} W^k_{\nu}.
\]

- **Observation of the weak \( W^i_\mu \)-bosons:**
  - The **experimentally observed \( W^\pm_\mu \)-bosons** are defined by
    \[
    W^\pm_\mu = \frac{1}{2} \left( W^1_\mu \mp i W^2_\mu \right).
    \]
  - The \( W^3_\mu \)-boson cannot be identified to the \( Z^0 \) or \( \gamma \): Both couple to left-handed and right-handed leptons.

\( SU(2)_L \) gauge theory cannot explain all data...
Summary - A gauge theory for weak interactions.

A gauge theory for weak interactions.

- Based on the non-Abelian $SU(2)_L$ gauge group.
- Matter (1): doublets with the left-handed component of the fields.
  - * Fundamental representation.
  - * Generators: Pauli matrices (over two).
- Matter (2): the right-handed component of the fields are singlet.
- Three massless gauge bosons.
  - * $(W^1_\mu, W^2_\mu) \implies (W^+_\mu, W^-_\mu)$.
  - * $W^3_\mu \neq Z^0_\mu, A_\mu$ ⇒ need for another theory: the electroweak theory.
Outline.


2. Special relativity and gauge theories.
   - Action and symmetries.
   - Poincaré and Lorentz algebras and their representations.
   - Relativistic wave equations.

   - Quantum Electrodynamics (QED).
   - Scattering theory - Calculation of a squared matrix element.
   - Weak interactions.
   - The electroweak theory.
   - Quantum Chromodynamics.

   - The Standard Model: advantages and open questions.
   - Grand unified theories.
   - Supersymmetry.
   - Extra-dimensional theories.
   - String theory.

5. Summary.
The electroweak theory (1).

Electromagnetism and weak interactions:

* $SU(2)_L$: **what is the neutral boson** $W^3$?
* How to get a **single formalism** for electromagnetic and weak interactions?

Idea: introduction of the hypercharge Abelian group:

* $U(1)_Y$: we have a **neutral gauge boson** $B \Rightarrow B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.
* $U(1)_Y$: we have a **coupling constant** $g_Y$.
* $SU(2)_L \times U(1)_Y$: $W^3$ and $B$ **mix** to the $Z^0$-boson and the photon.

Quantum numbers under the electroweak gauge group:

* $SU(2)_L$: left-handed quarks and leptons $\Rightarrow 2$.
* $SU(2)_L$: right-handed quarks and leptons $\Rightarrow 1$.
* $U(1)_Y$: fixed in order to reproduce the correct electric charges.
The electroweak theory (2).

Nœther procedure leads to the following Lagrangian.

\[ \mathcal{L}_{\text{EW}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_i^\mu W_i^{\mu\nu} \]

\[ + \sum_{f=1}^{3} \left[ \bar{L}_f \left( i \gamma^\mu D_\mu \right) L^f + \bar{e}_R \left( i \gamma^\mu D_\mu \right) e_R^f \right] \]

\[ + \sum_{f=1}^{3} \left[ \bar{Q}_f \left( i \gamma^\mu D_\mu \right) Q^f + \bar{u}_R \left( i \gamma^\mu D_\mu \right) u_R^f + \bar{d}_R \left( i \gamma^\mu D_\mu \right) d_R^f \right] . \]

* We have introduced the **left-handed lepton and quark doublets**

\[ L^1 = \begin{pmatrix} \Psi_{\nu e, L} \\ \Psi_{e, L} \end{pmatrix}, \quad L^2 = \begin{pmatrix} \Psi_{\nu \mu, L} \\ \Psi_{\mu, L} \end{pmatrix}, \quad L^3 = \begin{pmatrix} \Psi_{\nu \tau, L} \\ \Psi_{\tau, L} \end{pmatrix}, \]

\[ Q^1 = \begin{pmatrix} \Psi_{u, L} \\ \Psi_{d, L} \end{pmatrix}, \quad Q^2 = \begin{pmatrix} \Psi_{c, L} \\ \Psi_{s, L} \end{pmatrix}, \quad Q^3 = \begin{pmatrix} \Psi_{t, L} \\ \Psi_{b, L} \end{pmatrix}. \]

* We have introduced the **right-handed lepton and quark singlets**

\[ e_R^1 = \Psi_{e, R}, \quad e_R^2 = \Psi_{\mu, R}, \quad e_R^3 = \Psi_{\tau, R}, \]

\[ u_R^1 = \Psi_{u, R}, \quad u_R^2 = \Psi_{c, R}, \quad u_R^3 = \Psi_{t, R}, \quad d_R^1 = \Psi_{d, R}, \quad d_R^2 = \Psi_{s, R}, \quad d_R^3 = \Psi_{b, R}. \]
The electroweak theory (3).

The covariant derivatives are given by

\[ D_\mu = \partial_\mu - ig_\gamma Y B_\mu - ig_w T^i W_{\mu i} \]

- \( Y \) is the hypercharge operator (to be defined).
- The representation matrices \( T^i \) are \( \frac{\sigma^i}{2} \) and 0 for doublets and singlets.
Gauge boson mixing.

- **The neutral gauge bosons mix as**

\[
\begin{pmatrix}
A_\mu \\
Z_\mu
\end{pmatrix} = \begin{pmatrix}
\cos \theta_w & \sin \theta_w \\
-\sin \theta_w & \cos \theta_w
\end{pmatrix} \begin{pmatrix}
B_\mu \\
W_\mu^3
\end{pmatrix}.
\]

where the weak mixing angle $\theta_w$ will be defined later [see below...].

- **The neutral interactions (for the electron) are given by**

\[
L_{\text{int}} = \bar{L}e\gamma^\mu \left( g_Y Y_L e B_\mu + g_w \frac{\sigma^3}{2} W_\mu^3 \right) L_e + \bar{e}_R f \gamma^\mu g_Y Y_e R e_R f
\]

\[
= \bar{L}e\gamma^\mu \left( \cos \theta_w g_Y Y_L e + \sin \theta_w g_w \frac{\sigma^3}{2} \right) A_\mu L_e + \bar{e}_R f \gamma^\mu \cos \theta_w g_Y Y_e R A_\mu e_R f
\]

\[
+ \bar{L}e\gamma^\mu \left( -\sin \theta_w g_Y Y_L e + \cos \theta_w g_w \frac{\sigma^3}{2} \right) Z_\mu L_e - \bar{e}_R f \gamma^\mu \sin \theta_w g_Y Y_e R Z_\mu e_R f.
\]

- **To reproduce electromagnetic interactions, we need**

\[
e = g_Y \cos \theta_w = g_w \sin \theta_w \quad \text{and} \quad Q = Y + T^3.
\]

This defines the hypercharge quantum numbers.
Field content of the electroweak theory.

<table>
<thead>
<tr>
<th>Field</th>
<th>$SU(2)_L$ rep.</th>
<th>Quantum numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L^f_\nu$</td>
<td>2</td>
<td>$Y$: $-\frac{1}{2}$, $T^3$: $\frac{1}{2}$, $Q$: 0</td>
</tr>
<tr>
<td>$L^f_e$</td>
<td>2</td>
<td>$Y$: $-\frac{1}{2}$, $T^3$: $-\frac{1}{2}$, $Q$: $-1$</td>
</tr>
<tr>
<td>$e^f_L$</td>
<td>1</td>
<td>$Y$: $-1$, $T^3$: 0, $Q$: $-1$</td>
</tr>
<tr>
<td>$Q^f_\nu$</td>
<td>2</td>
<td>$Y$: $\frac{1}{6}$, $T^3$: $\frac{1}{2}$, $Q$: $\frac{2}{3}$</td>
</tr>
<tr>
<td>$Q^f_e$</td>
<td>2</td>
<td>$Y$: $\frac{1}{6}$, $T^3$: $-\frac{1}{2}$, $Q$: $-\frac{1}{3}$</td>
</tr>
<tr>
<td>$Q^f_u$</td>
<td>1</td>
<td>$Y$: $\frac{2}{3}$, $T^3$: 0, $Q$: $\frac{2}{3}$</td>
</tr>
<tr>
<td>$Q^f_d$</td>
<td>1</td>
<td>$Y$: $-\frac{1}{3}$, $T^3$: 0, $Q$: $-\frac{1}{3}$</td>
</tr>
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</table>
Electroweak symmetry breaking (1).

- The weak $W^{\pm}$-bosons and $Z^0$-bosons are observed as massive.
  * The electroweak symmetry must be broken.
  * The photon must stay massless.

- Breaking mechanism: we introduce a Higgs multiplet $\varphi$.
  * We need to break $SU(2)_L \Rightarrow \varphi$ cannot be an $SU(2)_L$-singlet.
  * The $Z^0$-boson is massive $\Rightarrow U(1)_Y$ must be broken $\Rightarrow Y \varphi \neq 0$.
  * $U(1)_{e.m.}$ is not broken $\Rightarrow$ one component of $\varphi$ is electrically neutral.

- We introduce a Higgs doublet of $SU(2)_L$ with $Y \varphi = 1/2$.

$$\varphi = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \equiv \begin{pmatrix} h_1^+ \\ h_2^0 \end{pmatrix}.$$

- The Higgs Lagrangian is given by.

$$L_{\text{Higgs}} = D_\mu \varphi^\dagger D^\mu \varphi + \mu^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^2 = D_\mu \varphi^\dagger D^\mu \varphi - V(\varphi, \varphi^\dagger).$$

* The covariant derivative reads $D_\mu \varphi = \left( \partial_\mu - i/2 g \gamma B_\mu - i g_w \sigma^i/2 W_{\mu i} \right) \varphi$.

* The scalar potential is required for symmetry breaking.
Electroweak symmetry breaking (2).

- **At the minimum of the potential, the neutral component of** $\varphi$ **gets a vev.**
  
  $$\langle \varphi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. $$

- **We select the so-called unitary gauge.**
  
  $$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}.$$

* The three Goldstone bosons have been eliminated from the equations. They have been **eaten by the** $W^\pm$ **and** $Z^0$ **bosons** to get massive.

* The remaining degree of freedom is the **(Brout-Englert-)Higgs boson.**
Mass eigenstates - gauge boson masses (1).

We shift the scalar field by its vev.

\[ \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + h \end{pmatrix}. \]

- The Higgs covariant derivative reads then:

\[
D_\mu \varphi = \frac{1}{\sqrt{2}} \partial_\mu \begin{pmatrix} 0 \\ \nu + h \end{pmatrix} - \frac{i}{\sqrt{2}} \begin{pmatrix} g_Y B_\mu + \frac{g_w}{2} W_3^{\mu} & \frac{g_w}{2} \left( W_1^{\mu} - iW_2^{\mu} \right) \\ \frac{g_w}{2} \left( W_1^{\mu} + iW_2^{\mu} \right) & \frac{g_Y}{2} B_\mu - \frac{g_w}{2} W_3^{\mu} \end{pmatrix} \begin{pmatrix} 0 \\ \nu + h \end{pmatrix}.
\]

- From the kinetic terms, one obtains the mass matrix, in the \( W^3 - B \) basis.

\[
D_\mu \varphi^\dagger D^\mu \varphi \to \begin{pmatrix} W_3^{\mu} & B_\mu \end{pmatrix} \begin{pmatrix} \frac{1}{4} g_w^2 \nu^2 & -\frac{1}{4} g_Y g_w \nu^2 \\ -\frac{1}{4} g_Y g_w \nu^2 & \frac{1}{4} g_Y^2 \nu^2 \end{pmatrix} \begin{pmatrix} W_3^{\mu} \\ B_\mu \end{pmatrix}.
\]

* The physical states correspond to eigenvectors of the mass matrix.

* The mass matrix is diagonalized after the rotation

\[
\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_w & \sin \theta_w \\ -\sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W_3^{\mu} \end{pmatrix},
\]

with \( \cos^2 \theta_w = \frac{g_w^2}{g_w^2 + g_Y^2} \).
Mass eigenstates - gauge boson masses (2).

- The mass matrix is diagonalized after the rotation

\[
\begin{pmatrix}
A_\mu \\
Z_\mu
\end{pmatrix} = \begin{pmatrix}
\cos \theta_w & \sin \theta_w \\
-\sin \theta_w & \cos \theta_w
\end{pmatrix}
\begin{pmatrix}
B_\mu \\
W^3_\mu
\end{pmatrix}.
\]

- As for the weak theory, we rotate \(W^1_\mu\) and \(W^2_\mu\).

\[
W^\pm_\mu = \frac{1}{2} (W^1_\mu \mp iW^2_\mu).
\]

- After the two rotations, the Lagrangian reads

\[
D^\mu \varphi^\dagger D_\mu \varphi = \frac{e^2 v^2}{4\sin^2 \theta_w} W^+_\mu W^-_\mu + \frac{e^2 v^2}{8\sin^2 \theta_w \cos^2 \theta_w} Z_\mu Z^\mu + \ldots.
\]

* We obtain a \(W^\pm\)-boson mass term, \(m_w = \frac{ev}{2\sin \theta_w}\).
* We obtain a \(Z^0\)-boson mass term, \(m_z = \frac{ev}{2\sin \theta_w \cos \theta_w}\).
* The photon remains massless, \(m_\gamma = 0\).
The Higgs kinetic and gauge interaction terms lead to

\[
D_\mu \phi^\dagger D^\mu \phi = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{e^2 v^2}{4 \sin^2 \theta_w} W_\mu^+ W^{-\mu} + \frac{e^2 v^2}{8 \sin^2 \theta_w \cos^2 \theta_w} Z_\mu Z^\mu \\
+ \frac{e^2 v}{2 \sin^2 \theta_w} W_\mu^+ W^{-\mu} h + \frac{e^2 v}{4 \sin^2 \theta_w \cos^2 \theta_w} Z_\mu Z^\mu h \\
+ \frac{e^2}{4 \sin^2 \theta_w} W_\mu^+ W^{-\mu} hh + \frac{e^2}{8 \sin^2 \theta_w \cos^2 \theta_w} Z_\mu Z^\mu hh.
\]

* We obtain **gauge boson mass terms**.
* We obtain a Higgs **kinetic term**.
* We obtain **trilinear interaction terms**.
* We obtain **quartic interaction terms**.
* Remark: no interaction between the Higgs boson and the photon.
Mass eigenstates - fermion masses (1).

- **The fermion masses are obtained from the Yukawa interactions.**

\[
\mathcal{L}_{\text{Yuk}} = -\bar{u}_{R} y_{u} (Q \cdot \varphi) - \bar{d}_{R} y_{d} (\varphi^\dagger Q) - \bar{e}_{R} y_{e} (\varphi^\dagger L) + \text{h.c.}
\]

* We have introduced the $SU(2)$ invariant product $A \cdot B = A_1 B_2 - A_2 B_1$.
* Flavor (or generation) indices are understood:

\[
\bar{d}_{R} y_{d} (\varphi^\dagger Q) \equiv \sum_{f,f'=1}^{3} \bar{d}_{R f'} (y_{d})_{f f'} (\varphi^\dagger Q_{f}).
\]

* The Lagrangian terms are **matrix products in flavor space**.

- **The mass matrices read**

\[
\mathcal{L}_{\text{mass}} = -\frac{\nu}{\sqrt{2}} \bar{u}_{R} y_{u} u_{L} - \frac{\nu}{\sqrt{2}} \bar{d}_{R} y_{d} d_{L} - \frac{\nu}{\sqrt{2}} \bar{e}_{R} y_{e} e_{L} + \text{h.c.},
\]

where we have performed the shift $\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + h \end{pmatrix}$, and introduced $u_{f}^{L} = \Psi_{u_{f},L}, \ldots$
Mass eigenstates - fermion masses (2).

- **The fermion mass Lagrangian read:**

\[
\mathcal{L}_{\text{mass}} = -\frac{v}{\sqrt{2}} \bar{u}_R y_u u_L - \frac{v}{\sqrt{2}} \bar{d}_R y_d d_L - \frac{v}{\sqrt{2}} \bar{e}_R y_e e_L + \text{h.c.}
\]

* The physical states correspond to eigenvectors of the mass matrices.

* Diagonalization: any complex matrix fulfill

\[
y = V_R \tilde{y} U_L^\dagger,
\]

with \(\tilde{y}\) real and diagonal and \(U_L, V_R\) unitary.

- **Diagonalization of the fermion sector:** we got replacement rules,

\[
\begin{pmatrix}
u_L \\
c_L \\
t_L
\end{pmatrix}
\rightarrow
\begin{pmatrix}
u_L' \\
c_L' \\
t_L'
\end{pmatrix}

= U_L^\dagger u_L , \quad
\begin{pmatrix}
\bar{u}_R \\
\bar{c}_R \\
\bar{t}_R
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\bar{u}_R' \\
\bar{c}_R' \\
\bar{t}_R'
\end{pmatrix}

\rightarrow
\begin{pmatrix}
U_L^\dagger u_L \\
V_R^\dagger \tilde{y}_u u_L \\
V_R^\dagger \tilde{y}_d u_L \\
V_R^\dagger \tilde{y}_e u_L
\end{pmatrix}
\]

* The up-type quark mass terms become

\[
-\frac{v}{\sqrt{2}} \bar{u}_R y_u u_L \rightarrow -\frac{v}{\sqrt{2}} \bar{u}_R' (V_R^\dagger) [V_R^\dagger \tilde{y}_u (U_L^\dagger)] U_L^\dagger u_L' = -\frac{v}{\sqrt{2}} \bar{u}_R' \tilde{y} u_L'
\]

where \(u_L, u_R\) are gauge-eigenstates and \(u_L', u_R'\) mass-eigenstates.
Mass eigenstates - flavor and $CP$ violation.

- The neutral interactions are still diagonal in flavor space, e.g.,

\[
\mathcal{L}_{\text{int}} = \frac{2}{3} e \, \bar{u}_L \gamma^{\mu} A_{\mu} u_L \to \frac{2}{3} e \left[ \bar{u}_L' (U^u_L)^\dagger \right] \gamma^{\mu} A_{\mu} \left[ U^u_L u'_L \right] = \frac{2}{3} e \, \bar{u}_L' \gamma^{\mu} A_{\mu} u'_L .
\]

due to unitarity of $U^u_L$.

- The charged interactions are now non-diagonal in flavor space, e.g.,

\[
\mathcal{L}_{\text{int}} = \frac{e}{\sqrt{2} \sin \theta_W} \, \bar{u}_L \gamma^{\mu} W^+_\mu d_L \to \frac{e}{\sqrt{2} \sin \theta_W} \left[ \bar{u}'_L (U^u_L)^\dagger \right] \gamma^{\mu} W^+_\mu \left[ U^d_L d'_L \right] = \frac{e}{\sqrt{2} \sin \theta_W} \, \bar{u}'_L \left[ (U^u_L)^\dagger U^d_L \right] \gamma^{\mu} W^+_\mu d'_L .
\]

- Charged current interactions become proportionnal to the CKM matrix,

\[
V_{\text{CKM}} = (U^u_L)^\dagger U^d_L \quad \text{[Nobel prize, 2008]} .
\]

- One phase and three angles to parameterize a unitary $3 \times 3$ matrix.

$\Rightarrow$ Flavor and $CP$ violation in the Standard Model.
Summary - The electroweak theory.

The electroweak theory.

- Based on the $SU(2)_L \times U(1)_Y$ gauge group.
  - $SU(2)_L$: weak interactions, three $W^i$-bosons acting on left-handed fermions and on the Higgs field.
  - $U(1)_Y$: hypercharge interactions, one $B$-boson acting on both left- and right-handed fermions and on the Higgs field.

- The gauge group is broken to $U(1)_{e.m.}$.
  - The neutral component of the Higgs doublet gets a vev.
  - Hypercharge quantum numbers are chosen consistently.
    ⇒ The fields get the correct electric charge ($Q = T^3 + Y$).
  - $W^1$ and $W^2$ bosons mix to $W^\pm$.
  - $B$ and $W^3$ bosons mix to $Z^0$ and $\gamma$.

- Yukawa interactions with the Higgs field lead to fermion masses.

- Experimental challenge: the discovery of the Higgs boson.
Outline.


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   - Scattering theory - Calculation of a squared matrix element.
   - Weak interactions.
   - The electroweak theory.
   - Quantum Chromodynamics.

   - The Standard Model: advantages and open questions.
   - Grand unified theories.
   - Supersymmetry.
   - Extra-dimensional theories.
   - String theory.

5. Summary.
The $SU(3)_c$ gauge group.

**Discovery of the color quantum numbers.** [Barnes et al. (1964)]

* The predicted $|\Omega\rangle = |sss\rangle$ baryon is a spin 3/2 particle.
* The $|\Omega\rangle$ wave function is **fully symmetric** (spin + flavor).
* This contradicts the spin-statistics theorem.

**Introduction of the color quantum number.**

**The $SU(3)_c$ gauge group.**

* Observed particles are **color neutral**.
* The **minimal** way to write an **antisymmetric wave function** for $|\Omega\rangle$ is

$$|\Omega\rangle = \epsilon_{mnl} |s^m s^n s^\ell\rangle .$$

* The quarks lie thus in a $\bar{3}$ of the new gauge group $\Rightarrow SU(3)_c$. 
Field content of the Standard Model and representation.

<table>
<thead>
<tr>
<th>Field</th>
<th>$SU(3)_c$ rep.</th>
<th>$SU(2)_L$ rep.</th>
<th>$U(1)_Y$ charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_f$</td>
<td>1</td>
<td>2</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>$e_{Rf}$</td>
<td>1</td>
<td>1</td>
<td>$-1$</td>
</tr>
<tr>
<td>$Q_f$</td>
<td>3</td>
<td>2</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$u_{Rf}$</td>
<td>3</td>
<td>1</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>$d_{Rf}$</td>
<td>3</td>
<td>1</td>
<td>$-\frac{1}{3}$</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>1</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$B$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$W$</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>$g$</td>
<td>8</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

* The matter Lagrangian involves the covariant derivative

$$D_\mu = \partial_\mu - ig_Y YB_\mu - ig_w T^i W^i_{\mu} - ig_s T^a g_{\mu a}.$$  

* We introduce a kinetic term for each gauge boson.

* The Higgs potential and Yukawa interactions are as in the electroweak theory.
Asymptotic freedom & confinement.

- **Based on the color** $SU(3)_c$ **interactions.**
  - * **The partons** (quarks, antiquarks, gluons) are colored.
  - * Observable particles (mesons $q\bar{q}'$, baryons $qq'q''$) are color neutral.

- **Running coupling constants.**
  - * Weak and electromagnetic: stronger at high energies (small distances).
  - * Strong: stronger at low energies (large distances).

- **High energies: asymptotic freedom** [Gross, Politzer, Wilczek, Nobel prize 2004].
  - * Small value for $g_s \Rightarrow$ perturbative calculations as series of $g_s$.
  - * The partons behave as free particles.
  - * In hadron collisions at high energies (e.g., LHC): Partons are interacting, not hadrons.

- **Low energies: confinement.**
  - * Large value of $g_s \Rightarrow$ partons are confined into hadrons.
  - * Non-perturbative physics $\Rightarrow$ models, experimental fits, ...
  - * Hadronization at low energies $\Rightarrow$ jets.
  - * In hadron collisions at high energies (e.g., LHC): One observes hadrons, not partons.
Quarks and gluons are not seen in Nature due to confinement.

In a hadron collision at high energy, they do interact (asymptotic freedom).

Predictions can be made thanks to the QCD factorization theorem.

\[
\frac{d\sigma_{\text{hadr}}}{d\omega} = \sum_{ab} \int dx_a \, dx_b \, f_{a/A}(x_a; \mu_F) \, f_{b/B}(x_b; \mu_F) \frac{d\sigma_{\text{part}}}{d\omega}(x_a, x_b, p_a, p_b, \ldots, \mu_F),
\]

where \( \sigma_{\text{hadr}} \) is the hadronic cross section (hadrons \( \rightarrow \) any final state).

* \( \sum_{ab} \) ⇒ all partonic initial states (partons \( a, b = q, \bar{q}, g \)).

* \( x_a \) is the momentum fraction of the hadron \( A \) carried by the parton \( a \).

* \( x_b \) is the momentum fraction of the hadron \( B \) carried by the parton \( b \).

* If the final state contains any parton: Fragmentation functions (from partons to observable hadrons).
Hadron collision - QCD factorization theorem (2)

• Predictions can be made thanks to the QCD factorization theorem.

\[
\frac{d\sigma_{\text{hadr}}}{d\omega} = \sum_{ab} \int d\lambda_{a} d\lambda_{b} f_{a/A}(x_{a}; \mu_{F}) f_{b/B}(x_{b}; \mu_{F}) \frac{d\sigma_{\text{part}}}{d\omega}(x_{a}, x_{b}, p_{a}, p_{b}, \ldots, \mu_{F}) .
\]

* \(f_{a/p_{1}}(x_{a}; \mu_{F}), f_{b/p_{2}}(x_{b}; \mu_{F})\) : parton densities.
  ◊ Long distance physics.
  ◊ ‘Probability’ to have a parton with a momentum fraction \(x\) in a hadron.

* \(d\sigma_{\text{part}}\): differential partonic cross section (which you can now calculate).
  ◊ Short distance physics.

\(\mu_{F}\) - Factorization scale.
  (how to distinguish long and short distance physics).
Parton showering and hadronization

* At high energy, initial and final state partons \textit{radiate other partons}.

* Finally, very low energy partons \textit{hadronize}.
Summary - The real life of a collision at the LHC

[Stolen from Sherpa]
Outline.


2. Special relativity and gauge theories.
   - Action and symmetries.
   - Poincaré and Lorentz algebras and their representations.
   - Relativistic wave equations.

   - Quantum Electrodynamics (QED).
   - Scattering theory - Calculation of a squared matrix element.
   - Weak interactions.
   - The electroweak theory.
   - Quantum Chromodynamics.

   - The Standard Model: advantages and open questions.
   - Grand unified theories.
   - Supersymmetry.
   - Extra-dimensional theories.
   - String theory.

5. Summary.
The Standard Model of particle physics.

* Is a **mathematically consistent** theory.
* Is **compatible** with (almost) all experimental results [e.g., LEP EWWG].
Open questions.

* Why are there three families of quarks and leptons?
* Why does one family consist of \( \{Q, u_R, d_R; L, e_r\} \)?
* Why is the electric charge quantized?
* Why is the local gauge group \( SU(3)_c \times SU(2)_L \times U(1)_Y \)?
* Why is the spacetime four-dimensional?
* Why is there 26 free parameters?
* What is the origin of quark and lepton masses and mixings?
* What is the origin of CP violation?
* What is the origin of matter-antimatter asymmetry?
* What is the nature of dark matter?
* What is the role of gravity?
* Why is the electroweak scale (100 GeV) much lower than the Planck scale (\(10^{19}\) GeV)?
# Outline

1. **Context.**
2. **Special relativity and gauge theories.**
   - Action and symmetries.
   - Poincaré and Lorentz algebras and their representations.
   - Relativistic wave equations.
3. **Construction of the Standard Model.**
   - Quantum Electrodynamics (QED).
   - Scattering theory - Calculation of a squared matrix element.
   - Weak interactions.
   - The electroweak theory.
   - Quantum Chromodynamics.
4. **Beyond the Standard Model of particle physics.**
   - The Standard Model: advantages and open questions.
   - **Grand unified theories.**
   - Supersymmetry.
   - Extra-dimensional theories.
   - String theory.
5. **Summary.**
Grand Unified Theories: a unified gauge group (1).

- **Can we reduce the arbitrariness in the Standard Model?**
  * A **single direct factor** for the gauge group.
  * A **common representation** for quarks and leptons.
  * **Unification of** $g_Y$, $g_W$ and $g_s$ **to** a single coupling constant.

- **Unification of the Standard Model coupling constants.**
  * The coupling constants (at zero energy) are **highly different**.

<table>
<thead>
<tr>
<th>Electromagnetism</th>
<th>Weak</th>
<th>Strong</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sim 1/137$</td>
<td>$\sim 1/30$</td>
<td>$\sim 1$</td>
</tr>
</tbody>
</table>

* The coupling strengths depend on the energy due to **quantum corrections**.

**Unification.**

- There exists a **unification scale**.
- The coupling strengths are **identical**.
Grand Unified Theories: a unified gauge group (2).

- **Running of the coupling constants.**
  * The coupling constant at first order of perturbation theory reads

\[
\frac{1}{\alpha} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} + \cdots
\]

* These calculations lead to the energy dependence of the couplings.

* Unification requires additional matter (e.g., supersymmetry [see below...]).
Grand Unified Theories: a unified gauge group (3).

- **Construction of a Grand Unified Toy Theory.**

- **How to choose of a grand unified gauge group.**
  
  * We want to pick up $G$ so that $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \subset G$.
  
  * **Electromagnetism must not be broken.**
  
  * **The Standard Model must be reproduced at low energy.**
  
  * Matter must be **chiral**.
  
  * Interesting cases are:

\[
G = \begin{cases} 
  \text{SU}(N) & \text{with } N > 4 \\
  \text{SO}(4N + 2) & \text{with } N \geq 2 \\
  E_6 
\end{cases}
\]

- **How to specify representations for the matter fields.**

  * **The Standard Model must be reproduced at low energy.**
  
  * The choice for the Higgs fields $\leftrightarrow$ **breaking mechanism**.

- **Specify the Lagrangian.**

\[
\mathcal{L}_{\text{GUT}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{breaking}}.
\]
Grand Unified Theories: a unified gauge group (4).

\[ \mathcal{L}_{\text{GUT}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{breaking}}. \]

* \( \mathcal{L}_{\text{kin}} \): Poincaré invariance.
* \( \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{gauge}} \): gauge invariance.
* \( \mathcal{L}_{\text{Yuk}} \): Yukawa interactions between Higgs bosons and fermions.
  ◊ Must be gauge-invariant.
  ◊ Fermion masses after symmetry breaking.
  ◊ Flavor and CP violation.
  ◊ Not obtained (in general) from symmetry principles.
* \( \mathcal{L}_{\text{breaking}} \): less known...
Grand Unified Theories: example of $SU(5)$ (1).

- **Gauge bosons**
  * A $5 \times 5$ matrix contains naturally $SU(3)$ and $SU(2)$.
    \[
    \begin{pmatrix}
    SU(3) & LQ \\
    LQ^\dagger & SU(2)
    \end{pmatrix} \in SU(5)
    \]
  * We have 12 additional gauge bosons, the so-called leptoquarks ($LQ$).
  * The matrix is traceless.
    $\sim$ The hypercharge is quantized.
    $\sim$ **The electric charge is quantized** ($Q = T^3 + Y$).

- $Y = \begin{pmatrix}
    \frac{1}{3} & 0 & 0 & 0 & 0 \\
    0 & \frac{1}{3} & 0 & 0 & 0 \\
    0 & 0 & \frac{1}{3} & 0 & 0 \\
    0 & 0 & 0 & -\frac{1}{2} & 0 \\
    0 & 0 & 0 & 0 & -\frac{1}{2}
    \end{pmatrix} \sim Q = \begin{pmatrix}
    \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
    0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\
    0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & -1
    \end{pmatrix}$.

- This matches the quantum numbers of the **right-handed down antiquark** $d_R^c$ and the **left-handed lepton doublet** $L$. 
Grand Unified Theories: example of \( SU(5) \) (2).

- **Fermions**
  - *Fundamental representation* of \( SU(5) \): \( d^c_R \) and \( L \).
  - \( 10 \) representation (antisymmetric matrix) \( \equiv \) 10 degrees of freedom.
    \( \sim \) the rest of the matter fields (10 degrees of freedom).

\[
5 \equiv \begin{pmatrix} d^c_R \\ d^c_R \\ \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} (d^c_R)^r \\ (d^c_R)^g \\ (d^c_R)^b \end{pmatrix} \quad 10 \equiv \begin{pmatrix} 0 & (u^c_R)^b & -(u^c_R)^g & -(u_L)^r & -(d_L)^r \\ -(u^c_R)^b & 0 & (u^c_R)^r & -(u_L)^g & -(d_L)^g \\ (u^c_R)^g & -(u^c_R)^r & 0 & -(u_L)^b & -(d_L)^b \\ (u_L)^r & (u_L)^g & (u_L)^b & 0 & -e^c_R \\ (d_L)^r & (d_L)^g & (d_L)^b & e^c_R & 0 \end{pmatrix}
\]

- The embedding of the gauge boson into \( SU(5) \) is **easy**.
- The embedding of the fermion sector is **miraculous**.
Grand Unified Theories: example of $SU(5)$ (3).

- **Higgs sector**
  
  * Two Higgs fields are needed.
    - One to break $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$.
    - One to break $SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{e.m.}$.
  
  * The simplest choice.
    - One field in the $24$ representation (special unitary $5 \times 5$ matrix).
    - One field in the fundamental representation.

- **Advantages of $SU(5)$**
  
  * Unification of all the interactions within a simple gauge group.
  * Partial unification of the matter within two multiplets.
  * Electric charge quantization.

- **Problems specific to $SU(5)$**
  
  * Prediction of the proton decay (lifetime: $10^{31} - 10^{33}$ years).
  * Prediction of a magnetic monopole.
  * Other problems shared with the Standard Model (three families, etc.)

- **Matter content.**
  - The matter is unified within a **single multiplet**.
  - $SO(10)$ has an additional degree of freedom $\Rightarrow$ the right-handed neutrino.
  - Explanation for the **neutrino masses**.
  - $E_6$ contains several additional degrees of freedom $\Rightarrow$ the right-handed neutrino plus new particles (to be discovered...).

- **The breaking mechanism leads to additional $U(1)$ symmetries.**
  - The gauge boson(s) associated to these new $U(1)$ are called $Z'$ bosons.
  - Massive $Z'$ **resonances** are searched at colliders [see exercises classes].

  $$pp \rightarrow \gamma, Z, Z' + X \rightarrow e^+ e^- + X \text{ or } \mu^+ \mu^- + X .$$

- **Other specific advantages and problems.**
  - $E_6$ appears naturally in string theories.
  - There is still no explanation for, e.g., the number of families.
  - **Gauge coupling unification is impossible without additional matter** $\Rightarrow$ e.g., supersymmetry.
Summary - Grand Unified Theories.

- The Standard model gauge group, $SU(3)_c \times SU(2)_L \times U(1)_Y$, is embedded in a **unified gauge group**.
- **Common representations** are used for quarks and leptons.
- The Standard Model is reproduced at low energy.
- More or less complicated **breaking mechanism**.
- Examples: $SU(5)$, $SO(10)$, $E_6$, ...
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   - String theory.
5. Summary.
Supersymmetry: Poincaré superalgebra (1).

Ingredients leading to superalgebra/supersymmetry.

* We have two types of particles, fermions and bosons.
  ⇒ We have two types of conserved charges, $B$ and $F$.

* The composition of two symmetries is a symmetry.
  ⇒ This imposes relations between the conserved charges.

\[
\begin{align*}
[B_a, B_b] &= i f_{a b}^\ c B_c, \\
[B_a, F_i] &= R_{a i}^\ j F_j, \\
\{F_i, F_j\} &= Q_{ij}^\ a B_a.
\end{align*}
\]
Supersymmetry: Poincaré superalgebra (2).

- **The Coleman-Mandula theorem (1967).**
  * The symmetry generators are assumed **bosonic**.
  * The only possible symmetry group in Nature is
    \[ G = \text{Poincaré} \times \text{gauge symmetries} \, . \]

- **Spacetime symmetries:** Poincaré
  \[
  \begin{align*}
  [L^{\mu\nu}, L^{\rho\sigma}] &= -i \left( \eta^{\nu\sigma} L^{\rho\mu} - \eta^{\mu\sigma} L^{\rho\nu} + \eta^{\nu\rho} L^{\mu\sigma} - \eta^{\mu\rho} L^{\nu\sigma} \right), \\
  [L^{\mu\nu}, P^{\rho}] &= -i \left( \eta^{\nu\rho} P^\mu - \eta^{\mu\rho} P^\nu \right), \\
  [P^\mu, P^\nu] &= 0 ,
  \end{align*}
  \]

- **Internal gauge symmetries:** compact **Lie algebra**.
  \[
  \begin{align*}
  [T_a, T_b] &= if_{abc} T_c \, , \\
  [P_\mu, T_a] &= 0 \quad \text{and} \quad [L_{\mu\nu}, T_a] = 0 .
  \end{align*}
  \]

- **The Haag-Łopuszański-Sohnius theorem (1975).**
  * Extension of the Coleman-Mandula theorem.
  * **Fermionic generators** are included.
  * The minimal choice consists in a set of **Majorana spinors** \((Q, \overline{Q})\).
  * \(N = 1\) **supersymmetry:** one single supercharge \(Q\).
Supersymmetry: Poincaré superalgebra (3).

The Poincaré superalgebra.

- **Spacetime symmetries.**
  \[
  \begin{align*}
  [L^\mu\nu, L^{\rho\sigma}] &= -i \left( \eta^{\nu\sigma} L^{\rho\mu} - \eta^{\mu\sigma} L^{\rho\nu} + \eta^{\nu\rho} L^{\mu\sigma} - \eta^{\mu\rho} L^{\nu\sigma} \right), \\
  [L^\mu\nu, P^\rho] &= -i \left( \eta^{\nu\rho} P^\mu - \eta^{\mu\rho} P^\nu \right), \\
  [P^\mu, P^\nu] &= 0,
  \end{align*}
  \]

- **Gauge symmetries.**
  \[
  \begin{align*}
  [T_a, T_b] &= if_{abc} T_c, \\
  [P_\mu, T_a] &= 0 \quad \text{and} \quad [L^\mu\nu, T_a] = 0.
  \end{align*}
  \]

- **Supersymmetry.**
  \[
  \begin{align*}
  [L^\mu\nu, Q_\alpha] &= \left( \sigma^\mu\nu \right)_{\alpha\beta} Q_\beta, \quad Q \text{ is a left-handed spinor}, \\
  [L^\mu\nu, \bar{Q}^\dot{\alpha}] &= \left( \bar{\sigma}^\mu\nu \right)_{\dot{\alpha}\dot{\beta}} \bar{Q}^\dot{\beta}, \quad \bar{Q} \text{ is a right-handed spinor}, \\
  [Q_\alpha, P_\mu] &= \left[ \bar{Q}^\dot{\alpha}, P^\mu \right] = 0, \\
  \{Q_\alpha, \bar{Q}_\dot{\alpha}\} &= 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu, \quad \{Q_\alpha, Q_\beta\} = \{\bar{Q}^\dot{\alpha}, \bar{Q}^\dot{\beta}\} = 0, \\
  [Q_\alpha, T_a] &= [Q_\alpha, T_a] = 0 \quad (Q, \bar{Q}) \text{ is a gauge singlet}.
  \end{align*}
  \]
Supersymmetry: Poincaré superalgebra (4).

**Consequences and advantages.**

* The supercharge operators *change the spin* of the fields.

\[ Q |\text{boson}\rangle = |\text{fermion}\rangle \quad \text{and} \quad Q |\text{fermion}\rangle = |\text{boson}\rangle . \]

* \((Q, \overline{Q})\) and \(P\) commute.
\[ \Rightarrow \text{fermions and bosons in a same multiplet have the same mass.} \]

\[ P^2 |\text{boson}\rangle = m^2 |\text{boson}\rangle \quad \text{and} \quad P^2 |\text{fermion}\rangle = m^2 |\text{fermion}\rangle . \]

* The composition of two supersymmetry operations is a *translation*.

\[ QQ + \overline{Q}Q \sim P. \]

* **Scalar masses** are protected from quantum corrections.

* It includes naturally *gravity*
  \[ \Rightarrow \text{New vision of spacetime} \Rightarrow \text{supergravity, superstrings.} \]

* **Unification** of the gauge coupling constants.
The Minimal Supersymmetric Standard Model.

- **Supersymmetry in particle physics.**
  - The Minimal Supersymmetric Standard Model: one *single supercharge*.
  - We associate one (new) superpartner to each Standard Model field.
    - Quarks ⇔ squarks.
    - Leptons ⇔ sleptons.
    - Gauge/Higgs bosons ⇔ gauginos/higgsinos.

- **Supersymmetry breaking.**
  - No scalar electron has been discovered.
  - No massless photino has been observed.
  - *etc.*

*Supersymmetry has to be broken.*
Supersymmetry breaking

- No supersymmetry discovery until now.

- **Supersymmetry breaking.**
  * **Superparticle masses shifted** to a higher scale.
  * **Breaking mechanism not fully satisfactory.**
  * Assumed to occur in a **hidden sector**.
  * **Mediated** through the visible sector via a given interaction.
  * **Examples:** minimal supergravity, gauge-mediated supersymmetry-breaking, etc..

- **Form of a supersymmetric Lagrangian.**
  \[
  \mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{break}}.
  \]
  * **\( \mathcal{L}_{\text{matter}} \):** supersymmetric gauge interactions for the matter sector.
  * **\( \mathcal{L}_{\text{gauge}} \):** supersymmetric gauge interactions for the gauge sector.
  * **\( \mathcal{L}_{\text{int}} \):** supersymmetric (non-gauge) interactions for the matter sector.
  * **\( \mathcal{L}_{\text{break}} \):** non-supersymmetric pieces of the Lagrangian (breaking).
  \[ \sim \mathcal{O}(100) \text{ new free parameters} \Rightarrow \text{universality assumptions} \sim \mathcal{O}(10). \]
Some supersymmetric phenomenology.

- **$R$-parity.**
  - $\mathcal{L}_{\text{int}}$: Under its general form:
    - $\leadsto$ lepton number violating interactions.
    - $\leadsto$ baryon number violating interactions.
    - $\leadsto$ proton decay.
  - Introduction of an *ad hoc* discrete symmetry, $R$-parity.
    - Standard Model fields: $R = +1$.
    - Superpartners: $R = -1$.
    - The problematic interactions are forbidden.

- **Consequences.**
  - The lightest superpartner (LSP) is stable.
    $\implies$ Cosmology: must be neutral and color singlet.
    $\implies$ Possible dark matter candidate.
  - Superparticles are produced in pairs.
    $\implies$ Cascade-decays to the LSP.
    $\implies$ Missing energy collider signature.
Summary - Supersymmetry.

- Extension of the Poincaré algebra to the Poincaré superalgebra.
- Introduction of supercharges.
- The Minimal Supersymmetric Standard Model: one single supercharge.
  * One superpartner for each Standard Model field.
  * LSP: possible dark matter candidate.
  * LSP: Collider signatures with large missing energy.
- More or less complicated breaking mechanism.
Outline.


2. Special relativity and gauge theories.
   - Action and symmetries.
   - Poincaré and Lorentz algebras and their representations.
   - Relativistic wave equations.

   - Quantum Electrodynamics (QED).
   - Scattering theory - Calculation of a squared matrix element.
   - Weak interactions.
   - The electroweak theory.
   - Quantum Chromodynamics.

   - The Standard Model: advantages and open questions.
   - Grand unified theories.
   - Supersymmetry.
   - Extra-dimensional theories.
   - String theory.

5. Summary.
Extra-dimensions in a nutshell (1).

- Main idea: the spacetime is not four-dimensional.

- Example: five dimensional scenario: $\mathbb{R}^4 \times \text{circle of radius } R$.
  
  * The fifth dimension is periodic.
  * Massless 5D-fields ⇒ tower of 4D-fields.

$$
\phi(x^\mu, y) = \sum_n \phi_n(x^\mu) \exp \left[ \frac{iny}{R} \right],
$$

where $y$ is the fifth-dimension coordinate.

* The 4D-fields $\phi_n$ are massive. Case of the scalar fields:
  
  ◊ We start from the Klein-Gordon equation in 5D

$$
\Box \phi(x^\mu, y) = [\Box - \partial_y^2] \phi(x^\mu, y) \implies [\Box + \frac{n^2}{R^2}] \phi_n(x^\mu) = 0
$$

* No observation of a Kaluza-Klein excitation $(\phi_n) \Rightarrow 1/R$ must be large.
Extra-dimensions in a nutshell (2).

- **Kaluza-Klein and unification.**
  - Basic idea: unification of electromagnetism and gravity (20's).
  - The 5D metric reads, with $M, N = 0, 1, 2, 3, 4$
    \[
    g_{MN} \sim \begin{pmatrix}
    g_{\mu\nu} & A_\mu = g_{4\mu} \\
    A_\mu = g_{4\mu} & \phi = g_{44}
    \end{pmatrix}.
    \]
  - 5D gravity $\sim$ 4D electromagnetism and gravity.

- **Extension to all interactions.**
  - The Standard Model needs 11 dimensions [Witten (1981)].
  - Problems with mirror fermions.

- **One possible viable model: Randall-Sundrum (1999).**
  - The Standard Model fields lie on a three-brane (a 4D spacetime).
  - Gravity lies in the bulk (all the 5D space).
  - The size of the extra-dimensions can be large (TeV scale).
  - KK-parity: dark matter candidate, missing energy signature, etc.

- **Other viable models are possible** (universal extra-dimensions, etc.).
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5. Summary.
String theory in a nutshell.

- **Point-like particles** ⇒ closed and/or open 1D-strings.
- **String propagation in spacetime** ⇒ a surface called a worldsheet.
- **The vibrations of the string** ⇒ elementary particles.
- **Worldsheet physics** ⇒ spacetime physics (quantum consistency).
  - 10-dimensional spacetime (**extra-dimensions**).
  - Extended gauge group (**Grand Unified Theories**).
  - Supergravity (**supersymmetry**).
- **Compactification from 10D to 4D**.
  - Must **reproduce the Standard Model**.
  - **Many possible solutions**.
  - **No solution found** so that all experimental constraints are satisfied.
Outline.

1 Context.

2 Special relativity and gauge theories.
   - Action and symmetries.
   - Poincaré and Lorentz algebras and their representations.
   - Relativistic wave equations.

3 Construction of the Standard Model.
   - Quantum Electrodynamics (QED).
   - Scattering theory - Calculation of a squared matrix element.
   - Weak interactions.
   - The electroweak theory.
   - Quantum Chromodynamics.

4 Beyond the Standard Model of particle physics.
   - The Standard Model: advantages and open questions.
   - Grand unified theories.
   - Supersymmetry.
   - Extra-dimensional theories.
   - String theory.

5 Summary.
Summary.

- The **Standard Model has been constructed from experimental input**.
  - Based on symmetry principle (relativity, gauge invariance).
  - Is consistent with quantum mechanics.
  - Is the most tested theory of all time.
  - Suffers from some limitations and open questions.

- Beyond the **Standard Model theories are built from theoretical ideas**.
  - Ideas in constant evolution.
  - Grand Unified Theories.
  - Supersymmetry.
  - Extra-dimensions.
  - String theory.
  - * etc..